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ABSTRACT

A pervasive theme running throughout a conference on mathematical education was the transformation of the learning and teaching of mathematics using technology. This publication is a collection of papers representing an American view of the events of the international conference. The papers share the views of collegiate mathematicians, elementary school teachers, teacher educators, researchers in mathematics education, middle and secondary school teachers, two-year-college faculty members, and others. The first three authors focus on the special aspects of the relationship between prior ICME meetings and higher education (S. Hill), technology and mathematics education (J. T. Fey), and assessment around the world (N. Webb). The remaining 35 papers report the activities, insights, and growth of many of the 40 individuals who received support awards for attending the Congress through a National Science Foundation grant to the National Council of Teachers of Mathematics. (MDH)

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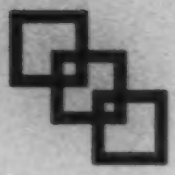

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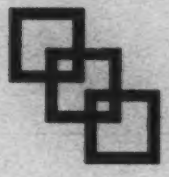

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AMERICAN PERSPECTIVES ON THE SEVENTH INTERNATIONAL CONGRESS ON MATHEMATICAL EDUCATION

Edited by

John A. Dossey
Illinois State University

National Council of Teachers of Mathematics

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PREFACE

The Seventh International Congress on Mathematical Education (ICME-7) was held in Québec City, Québec, Canada, on 17–23 August 1992. The campus of the Université Laval provided a congenial atmosphere for the nearly 3000 mathematics educators from approximately 90 countries in attendance. The weather and the charm of Québec, as a province and a city, provided the backdrop for seven days of intensive discussions, presentations, and debates over the present status and future possibilities of mathematics education from a large number of perspectives.

Perhaps the most pervasive theme running through this conference was the transformation of the learning and teaching of mathematics being brought by technology. This was highlighted by the one-day miniconference on technology that opened ICME-7. However, this topic continued through papers on curriculum, teacher education, research, and beliefs. It was evident in the display areas and in the sidewalk and beer-tent discussions in the late afternoon. Close behind technology in attention was the theme of assessment. Discussions, even debates, took place over the appropriate forms for assessing student learning and progress in mathematics. Authentic assessment was the password and developing rubrics for evaluating performance tasks the work.

This publication is a collection of papers portraying an American view of the happenings of ICME-7. It shares the views of collegiate mathematicians, elementary school teachers, teacher educators, researchers in mathematics education, middle and secondary school teachers, two-year-college faculty members, and others. The first three authors, Hill, Fey, and Webb, were asked to focus on special aspects of the Congress in particular. The remaining papers report the activities, insights, and growth of many of the 40 individuals who received support awards for attending the Congress through a National Science Foundation grant to the National Council of Teachers of Mathematics. As such, the reports do not necessarily reflect the views of either the National Science Foundation or the National Council of Teachers of Mathematics.

The success of the Congress was greatly aided by its setting in Québec. The walled city, its warm and friendly inhabitants, the Plains of Abraham, the Saint Lawrence, and the province's rich, and international, history provided a perfect setting for investigating the role of culture and beliefs in the nurturing of mathematical understanding. The Canadian organizing committee for ICME-7 outdid itself in covering every detail to make the week a special memorance for all in attendance. ICME-8 will be held in Seville, Spain, in the summer of 1996.

In closing, I wish to thank Iris Carl, James Gates, Mary Lindquist, Henry Pollak, and Stephen Willoughby for their many contributions to the project, from the selection of awardees to their assistance in editing this publication. Special thanks also go to Marilyn Parmantie for her assistance in preparing the manuscript.

John Dossey
Project Director and International Representative
National Council of Teachers of Mathematics

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ICME-7 AND TERTIARY LEVEL MATHEMATICS: UNE PETITE AFFAIRE

Shirley Hill
University of Missouri, Kansas City, Missouri

The programmatic relationship between the International Congresses on Mathematical Education and higher education has rarely risen above the level of a flirtation. This "at arms length" interaction is particularly curious when we acknowledge that the large majority of attendees at these quadrennial conferences reside professionally at tertiary level institutions.

My estimate of ICME-7, however, is that the relationship has progressed to that of *une petite affaire*. Might we hope for, at some future time, a grand passion or even a marriage? The trend appears to be in that direction. Just as has been true in the United States for the past several years, there seems to be an increasing acknowledgment internationally that mathematics in higher education has at least its share of thorny problems and seriously demands attention and help. But also in parallel to concerns in the United States, the focus is extremely narrow, overwhelmingly on calculus.

In the opening plenary session at the Congress, Geoffrey Howson (UK) referred to the weakness of the relationship when he said that tertiary level education has received "little attention from mathematics education in these congresses." He pointed out that an unfortunate differentiation between teaching at tertiary and other levels is deeply embedded in our cultures, as shown by almost universally acknowledged differences in meaning between the terms *teacher* or *master* and *professor*. Howson contends that these distinctions can be seen to have many consequences, differences in resources, in expectations, in status, in aims, and in attitudes toward mathematics. The focus on "learning problems as something existing only at precollege level" reflects an assumption that has split our community, he stated.

Howson warned against the danger of partitioning the community into fragmented groups: mathematicians, mathematics educators, and mathematics teachers. Progress in mathematics education, he believes, depends on the coming together of the three parts of one community.

Howson also pointed out that the current call for "mathematics for *all*" has enormous implications for tertiary education. Employment demands mean a greater proportion of the student population in tertiary level mathematics courses, and this will necessitate a rethinking and a restructuring of the mathematics program in higher education. He sees everywhere in this restructuring a high potential for a clash between short-term political expediency and long-term planning.

In another plenary session, Colette LaBorde (FRA) gave a beautifully illustrated and persuasive argument for modeling in geometry using computer graphics. While some of her talk was targeted to pre-college level mathematics instruction, much of it had a direct message for collegiate teaching as well. She stressed that computer graphics presents a model and a model is the experimental laboratory for a reality field not bound by the limitations of the actual reality field. In some sense, it permits us to "do the impossible." Her focus was on transformational geometry as a dynamical model. Certainly, this true integration of computer graphics into instruction and her emphasis on visual representation as playing a key role in geometric reasoning have many direct implications for teacher education.

In addition to plenary sessions, there were several major, large-audience lectures. One of these was given by Miguel de Guzman (SPN), president of the International Commission on Mathematical Instruction. As a historian and philosopher, he presented a talk that could be called "foundational" and as such, it addressed all levels of education but especially the tertiary level. In discussing what attitudes to foster in students, he stressed the importance of both understanding and appreciating the theoretical underpinnings of mathematical problems. While each student needs a personal vision of science and mathematics, a knowledge of history is essential as a guide to the future and a guide to one's own research. The student needs to see the imprint of history on mathematics and the imprint of mathematics on mankind's history, according to de Guzman. History, he says, enables us to see mathematics as a human activity, therefore fallible and imperfect.

Uri Treisman's (USA) major lecture spoke directly to undergraduate departments of mathematics. He described his, by now well-known, program for minority students in mathematics. This successful program is based on the creation of what Treisman calls "rich communities in the center of mathematics departments." This was a key point for both mathematics faculty and for college and university administrations. Treisman contends that, even within the best-intentioned administrative programs to help students who are having difficulty, these students, and particularly minority students, are marginalized. He stated that these programs, such as in "learning centers" separate from the departments, are inevitably politicized, and if politicized, they will be marginalized.

Minorities, Treisman asserts, have been isolated traditionally. While they typically have a rich community and social life, they have remained isolated in their academic lives. It is, in Treisman's thesis, the role and responsibility of the mathematics departments and faculty of colleges and universities to create within academic departments the rich community that encourages and nurtures minority students. This should not be left to the politicized administrative programs such as those often called "learning centers."

Coming from a different vantage point, Treisman's lecture challenges traditional higher education assumptions about mathematics instruction and thus reinforces Howson's position about the necessity of rethinking and restructuring the tertiary level in mathematics education.

Teacher Education

The exception to the claim that the international congresses on mathematics have paid scant attention to higher education is, of course, teacher education. While the fraction of sessions devoted specifically to teacher education has been quite low, the centrality of teacher education to any attempts to address problems of mathematics education has been unchallenged. There have been, in all the congresses to date, working groups and other strands dedicated entirely to teacher education in mathematics.

The difficulty in attempting to report on tertiary level mathematics instruction at the congresses is that the direct focus on either undergraduate mathematics or teacher education is through the program component called the "working group." The problem is that working groups are scheduled entirely within the same time slots. So a difficult choice must be made. I chose to join the group "Undergraduate Mathematics for Different Groups of Students" precisely because I felt that attention to postsecondary mathematics is picking up and one might find a new slant on the problems of mathematics education not found in previous congress programs. At any rate, the readers of this report might be less aware of trends and concerns in this area newly awakened to a consciousness of a need for radical change than they are of what is happening in teacher education.

It is important to point out that, despite the title of this report, it cannot cover in any inclusive sense tertiary level mathematics education at ICME-7. From this point on it will focus on the work of the only program component targeted toward undergraduate mathematics.

Before turning exclusively to that topic, I will say a bit more about teacher education. Since I could not attend the sessions of the working group itself, I was curious to find out what impressions could be gained from perusing the titles and description of the presentations made to and by the group members and by reading the abstracts of all papers on teacher education. Were there common themes or discernible trends? Not really, although there remains a remarkably strong international consensus on the focus on problem solving as a primary goal, on the importance of modeling in both use of physical materials and through computers, and certainly on the grounding of instructional methodology in constructivism. The impression is clearly one of a "staying the previous course" and a strengthening of a consensus among those types who attend these congresses, rather than the emergence of new themes or ideas.

What struck me most forcefully as I studied the abstracts and presentation titles was the almost universal pattern of reporting on projects and experiments. Nearly every session and paper in the teacher education working group seems to have been about "what we did" or "what happened when we tried this." That made me wonder if this was common throughout the Congress program. In taking yet another and closer look at the program, I think a case could be made that it was. The Congress was much more about what "is" than what "ought to be." There is little discourse about the aims of mathematics education, its values and purposes. There is almost no challenge to the assumptions about why we teach mathematics. If there ever was a tension between the technocrats and the philosophers, then the technocrats have won out. This Congress, like most recent Congresses, is pretty much a reporting about the programs and projects around the world. Perhaps this is as it should be. Perhaps the intent is to be an international scientific

conference, but it isn't really that because little actual research is reported and there is little hint of competing theories. There is little to challenge.

The remarks above are not intended to be evaluative. I do not consider the state of affairs to be negative because I suspect that most attendees come precisely to find out what is going on among the neighbors on this planet. But it is true, I believe, that the issues that demand discussion and philosophical analysis are treated in large-group sessions, where no discussion is possible, whereas the smaller working groups, where discussion does take place, are largely reporting and informational sessions. It may be that the format needs rethinking, just to stir things up a bit.

Undergraduate Mathematics

The only working group focused solely on undergraduate mathematics was actually entitled "Undergraduate Mathematics for Different Groups of Students." It was intended to take a broader perspective than just the mainstream mathematics in mathematics departments. The range of papers and presentations was quite broad in fact. There was mathematics for scientists, mathematics for business students, mathematics in-service courses for the nonspecialist, mathematics for computer science, mathematics for economics, and mathematics for nurses.

As one might expect from the thrust of the topics above, most of the presentations and papers were about projects and programs at the presenters' institutions. The sessions, like those of the teacher education sessions, were primarily informational. Discussions tended to be more question-and-answer than exchanges of viewpoints. Various kinds of mathematics were represented, with several presentations and papers on discrete mathematics and on statistics. But the distinct impression in attending as many sessions as possible on the topic of undergraduate mathematics, both in this working group and other parts of the program, is that the major focus in attempts to reform undergraduate mathematics is on calculus. One participant in this working group commented to me, "There is certainly a lot of attention to calculus at this Congress." One can speculate on the reasons. Is it because calculus is still the common core of many programs? Is it because of the traditional centrality of calculus? Is it fashionable or trendy to experiment with calculus instruction? Is calculus where the worst teaching problems occur? Or the worst learning problems? Maybe it is all of the above. Maybe the focus isn't representative at all but is just the result of the high proportion of participants from the United States, where we have put most of the reform effort at the undergraduate level on calculus.

Without claiming that they are representative of all references to tertiary education at ICME-7, I have selected four papers to report on in some detail. All four were part of the working group program.

The first relates to a course in precalculus, a reform project in the United States funded by NSF. Project directors are B. A. Fusaro and Sheldon P. Gordon. The course has as its aim an increase in the number of students who go into the calculus portion of the "mathematics pipeline," or "filling the mathematics tank," as the authors put it. The course is intended to be a "dramatically different alternative to standard precalculus courses," and it emphasizes the qualitative, geometric, and computational aspects of mathematics within a framework of mathematical modeling.

The models are based primarily on difference equations, data analysis, probability, and matrix algebra. A thread interwoven throughout the course is the notion of probabilistic ideas in the context of performing random simulations. Data analysis techniques are introduced early. Matrix algebra is developed as a unifying tool for investigating a wide array of applications, including systems of difference equations, Markov processes, regression analysis, geometry, and networks. The course involves considerable computer or graphing calculator work.

The second paper is a presentation on "Mathematics for Scientists" by Fred Simons (NLD). Simons points out that service courses for nonmathematicians are a staple in nearly all universities around the world. Furthermore, he states that they have much in common: some calculus, some linear algebra. However, the content "dates from the times that all mathematical computations had to be done with pencil and paper and in fact they reflect the needs of scientists of decades ago." Further evidence of their obsolescence is that most exercises in undergraduate service courses "can be solved by pressing some keys of widely spread small computer algebra packages such as DERIVE."

Simons suggests that a starting point for the development of new courses could be that "what a computer can do better than we can, should be done by computers." But he also points out that for the efficient use of the really useful advanced programs one needs a thorough understanding of mathematics. He

concludes, therefore, that future courses for scientists should concentrate on mathematical insight "rather than on imitating gimmicks." From the beginning, he states, computer algebra packages should be used for doing the computation.

Simons confesses to some disappointment in the way computer algebra packages are being introduced into many courses today. Often it is just a matter of predicting "what the computer will show" and then using the computer "for a verification of what you have seen" and for finding "technical details such as zeros and extreme values."

A major point that Simons makes about mathematics courses in general, not just the service courses for scientists, is that the introduction in a serious way of computer use is far more than just doing what we have always done better and more efficiently with technology. Much more critically, as he maintains, the courses themselves and their entire content must change. "With a computer at hand," he states, "things that are considered to be very important in traditional courses might be less important or even unimportant in computer courses."

Simons' own experience suggests that "with computer courses good students become better and bad students worse than with traditional courses." The last is a startling statement and, if true, it seems to me that we need to do a careful examination of our aims and our basic assumptions about success in mathematics study.

The third paper I have chosen to report on is one given by László Máté (HUN). It describes a course for engineering students who are foreign students mainly from Middle Eastern countries. The course is called "Fractals and Dynamical Systems."

Máté's assumptions about the learning and the applying of mathematics ran up against some cultural differences. He believes that "visual intuition is essential in mathematical invention and hence also in real understanding." His course uses computer graphics to combine the visual with the experimental. He states that visual experiences and the use of the computer in addition to the deductive method is more effective than the purely traditional theorem-proof presentation. One argument for these multiple approaches is that they call upon a wider spectrum of student abilities. Thus, he believes, the practice and possibly the philosophy of mathematics and mathematics education is brought closer to that of the experimental sciences.

Máté goes on to concede that these assumptions about the efficacy of visual intuition require questioning when it comes to his current students, who are almost all from the Islamic culture. For them, he asserts, verbal communication is preferred. He proffers two possible reasons for their preference: (1) that their "real world" experiences are very different from those of Western culture, and (2) that their education has accustomed them to memorization from written texts.

Having introduced this intriguing challenge to his own situation, he does not go on to examine whether these students should be encouraged to develop more visual intuitions or whether his own assumptions about effective teaching and learning need to be analyzed further and possibly revised. The conflict he describes is, in my opinion, a deep and important issue, and I wished that the working group could have debated the issue at length. Unfortunately, the format and time constraints imposed by scheduling a large number of short presentations rarely affords the participants the luxury of a thorough discussion. As I mentioned earlier in this paper, I think that the label "working group" has become something of a misnomer at the Congresses.

In an exception to a point I made earlier about the absence of challenge to the purposes to be served by teaching mathematics, a paper by John Searl (SCO) begins by asking, "Why do we teach them mathematics?" The paper itself is entitled, "Developing Research into Mathematical Education in Higher Education." Searl's answer to his own question is that the usefulness of mathematics is "more akin to that of a language." He then discusses mathematics as a means of communication.

Then in a subsequent discussion of "what mathematics" to teach, he reviews Skemp's contrast between instrumental learning and relational understanding. He makes a case for why many teachers choose to focus on instrumental learning and states that relational understanding sometimes requires more course content than an already "overburdened syllabus" may tolerate. He then relates his discourse on learning difficulties back to language in a review of the idea of "procept" from studies by David Tall (UK). Tall means by "procept" procedures and concepts that share the same notation. Searl suggests that some difficulties may be inherent in mathematics and apparently believes that confusions in language are among them.

After a review of some promising research approaches, Searl draws a rather lame conclusion to his paper. Innovations over the past thirty years have been carefully evaluated to remove blunders and improve

presentation. Thus the emphasis has been on the mechanisms of delivery rather than on the mathematical development of students. This assertion went without challenge in the working group, but I rather think that there are many people who would disagree with him and would be able to marshall evidence in support of their positions, although not in the format of the Congress because of the severe limitation on the time allotted to a working group paper.

All in all, there was a hint at ICME-7 that the grave problems that exist in tertiary level mathematics education may begin to gain the attention they need.

TECHNOLOGY AND MATHEMATICS EDUCATION AT ICME-7

James T. Fey
University of Maryland, College Park, Maryland

Uses of technology in mathematics education have been demonstrated and debated at international congresses for nearly twenty years. Not surprisingly, the range of ways that technology is being applied and the extent of interest in those applications has increased dramatically over the past ten years—stimulated not only by the availability of low-cost microcomputers and now graphing calculators but also by a striking increase in the development of interesting videos to support mathematics curricula and teaching.

North American mathematics educators who regularly attend professional meetings, like the NCTM regional and national conferences, take for granted broad access to sophisticated hardware and powerful software. For nearly a decade we have been exploring and debating the influence of such new tools on the goals and teaching approaches for our mathematics programs. The same active investigation of calculator and computer and video use has been conducted in several other highly developed countries—notably the United Kingdom, France, Sweden, and Israel—but in much of the rest of the world the exploration of technology has been more cautious. For example, Japanese schools have not, until very recently, made significant use of either calculators or computers in mathematics teaching at the school level. Many developing countries have simply not had the financial resources to provide school access to technology.

At ICME-5 in Adelaide in 1984, a small number of working groups gathered to hear reports of innovative work with technology from a few pioneers in the field, but there was very little hardware or software in evidence and most Congress participants were being exposed for the first time to the promise and challenge of computers. At ICME-6 in Budapest in 1988 there was a much more substantial strand of sessions related to calculators and computers at all levels from primary school through university mathematics education, and it was clear that development activity was spreading around the world (see Fey, 1989). Unfortunately, because of the difficulties of bringing hardware and software into Hungary at that time, there was very little working technology on display at the Budapest meeting. This all changed at the Quebec meeting in 1992. By any reasonable measure, participants at ICME-7 witnessed a significant escalation in the attention given to technologies and their impact on school and university mathematics around the world.

Technology in the ICME Program

ICME-7 began with some ceremonial sessions and one plenary lecture on the morning of the first day. However, the real scientific work of the Congress began later that day when the entire afternoon was devoted to a 3.5-hour miniconference on calculators and computers. Every participant in the Congress was invited to join one of five miniconference program strands focusing on 5- to 11-year-old students, 11- to 16-year-old students, 15- to 18-year-old students, mathematics undergraduates, or teacher education. Each program strand consisted of a one-hour plenary lecture/demonstration session followed by two 50-minute sessions offering parallel workshops and paper presentations, much like an NCTM meeting format. Each miniconference strand included presentations by an array of experts in the use of calculators or computers. In almost every session leaders used technology with participants much as they use it with students in their experiments—hands-on workshops with graphing calculators or in computer labs and lecture/demonstration sessions with a single computer and a larger audience. Presenters showed work from twenty countries, with innovative work in North America prominent in each strand of the program.

The miniconference made technology a prominent feature of the Congress right from the start, and this attention was continued throughout the week-long Congress program: in two special working groups—WG-16: "Impact of Calculators on Elementary School Curricula" and WG-17: "Technology in Service of the Mathematics Curriculum"—that met for four 1.5-hour sessions throughout the Congress; in a theme group, TG-13: "TV in the Mathematics Classroom"; in major lectures by Benoit Mandelbrot, Celia Hoyles, and Seymour Papert; and in countless informal software and video presentations at the exhibits and poster

displays. Furthermore, it is evident in the program for other working groups that sessions of those groups devoted considerable attention to the impact of technology in all aspects of mathematics education. For example, the working group on geometry included a keynote address by Paul Goldenberg that featured creative new video and computer software materials, a session on "Logo and a Young Child's Language of Space," as well as an entire subgroup that spent the week considering the impact of computers and conjecturing in secondary school geometry.

Influence of Technology on Mathematics Education

With the unmistakable evidence that calculators and computers have permeated thinking about all aspects of school mathematics curricula and teaching, it is reasonable to ask about the shape and extent of that influence—progress since the last ICME and prospects for the future. In 1989, writing in retrospect on the Budapest ICME, Richard Shumway proposed ten categories of uses for computers and calculators in mathematics education: (1) computation, (2) graphics, (3) simulations, (4) programming, (5) drill and practice, (6) teacher utility, (7) information management, (8) tutorial, (9) concept learning, and (10) problem solving. He observed that as computer use in education evolved from its origins in about 1965, activity "[began with] emphasis on computations and programming, moved to drill and practice, teacher utility, information management, and tutorial uses, and [in 1988] seem[ed] to focus on computations, graphics, simulations, concept learning, and problem solving."

Although it is clearly impossible for one observer to reliably characterize the content of a program strand with as many threads as technology at ICME-7, the sessions I attended and the informal conversations I had with participants from many countries suggest that Shumway's summary describes the situation four years later as well. In the miniconference, working groups, and special lectures there was very little evidence of interest in computer programming as a vehicle for learning mathematics, in computer tutors that control the learning environment as electronic instructors in drill-and-practice software, or in the variety of ways that computers can be used to assist with information management tasks of teaching. The strongest theme in most sessions was the search for powerful computer and calculator tools that would enable students to conduct mathematical investigations that solve important real problems and yield understanding of important concepts. My impressions are based most specifically on the miniconference, especially strand MC-3: Ages 15–18 for which I was the organizer, and on the working group that examined the role of computers in geometry.

Technology-based innovation in mathematics teaching is driven by at least three kinds of development: (1) new hardware that makes computational power more readily accessible to teachers and students; (2) new software that applies computational power to mathematically interesting tasks; and (3) new conceptions of curriculum and teaching that show ways to take advantage of computational power for teaching, learning, and problem solving.

Trends in Computational Hardware and Software

The well-known North American fascination with high technology tools for tasks in a broad array of practical, scientific, technical, and entertainment endeavors means that new hardware and software applicable to mathematics education is usually available here as soon as it is developed. At ICME-6 in Budapest, cutting-edge mathematics educators were already demonstrating and sharing results of their classroom experiments with graphing calculators, computer algebra systems, and geometry construction/measurement tools. Much of the development in the four years since that meeting has been refinement and extension of those and other standard calculator and computer tools—making them more powerful and easier for students to use.

As mathematics educators think about new directions for curriculum and teaching, they have to consider an impressive array of tools. For numerical operations there are inexpensive and powerful calculators with multiline displays, operations on common fractions and matrices, and easy computation of iterative processes. For graphic operations there are graphing calculators and computer plotting software for two- and three-dimensional displays, as well as geometry exploration tools, like *Cabri-géomètre* and the Geometer's Sketchpad, that permit continuous transformation of shapes subject to prescribed invariant conditions, and a variety of software packages for producing fractal images. For algebraic operations there are powerful computer algebra systems like Mathematica, Maple, and Derive, and many of the formal symbolic

manipulation routines are now appearing in hand-held devices as well. A powerful array of statistical packages are available for data analysis and reasoning about probabilistic situations.

Trends in Curriculum and Instruction

Although many of the impressive hardware and software tools were available in 1988 at Budapest, the past four years have seen significant exciting work to apply the tools in curriculum and teaching experiments. I was able to follow closely reports of how this work is proceeding in geometry, through presentations in the miniconference and in the subgroup of the geometry working group that focused on computers and conjecturing. In those sessions the most frequently mentioned software was geometry drawing tools, especially the Geometric Supposer series, *Cabri-géomètre*, and the Geometer's Sketchpad. From people who had experimented with that software, I heard consistent reports of work that promised benefits in teaching and learning:

1. *Enhanced Student Learning*. Most curriculum developers and researchers using the geometry drawing/measuring tools suggest that student learning will be enhanced in these ways:
   a. Using the drawing tools to conduct independent investigations in geometry should help students develop the ability to plan and conduct open-ended explorations in search of interesting and significant patterns in families of shapes, and the *habit* of applying such strategic behavior in a variety of problem-solving situations. This experience with geometric investigations should also develop habits and abilities to apply powerful mathematical heuristics like testing special and extreme cases or systematically varying some factors as others are held constant. Several developers of geometry investigation curricula argue that those heuristic lessons are likely to transfer to other mathematical topics as well.
   b. Electronic assistance with drawing and manipulation of geometric configurations holds great promise in developing stronger student spatial visualization skills.
   c. Use of computer tools to conduct electronic drawing experiments should develop student understanding of the role of empirical work in formulating mathematical conjectures and arguments (Yerushalmy, 1990). This kind of empirical experience can sharpen understanding of geometry, in particular, as a source of models for aspects of visual observations and mathematics, in general, as a source of models for patterns in the world of number, shape, position, and motion.
   d. Because geometry exploration tools encourage formulation of conjectures based on investigation of several special cases, they lead students to a deeper and clearer understanding of the difference between inductive and deductive reasoning in geometry and in mathematics as a whole. They help students see better how a theorem is a general statement that summarizes a variety of instances of relationships, not a statement about a single static embodiment of hypotheses and conclusions. Especially in using the "drag mode" of Cabri or Sketchpad, students gain special insight into the notion of invariance under transformation (Schumann, 1991).
   e. Deductive proof is clearly the ultimate standard of verification for mathematical propositions. But in many cases such a proof shows only how one result is a necessary consequence of earlier assumptions and deductions; it does not provide intuitive understanding of why the pattern in focus holds. Contemporary tools for drawing and manipulating geometric figures help students gain that kind of deeper *understanding* that is so productive in applying knowledge to new situations. This contribution of technology to mathematical learning is, of course, also provided by numerical, graphic, and symbolic explorations in arithmetic, algebra, geometry, discrete mathematics, and statistics.
   f. The new geometry construction tools also help students depict and analyze complex geometric relationships that are generally inaccessible to students if they are provided with only paper and pencil and ruler and compass. For instance, the modern electronic drawing tools make it relatively easy to construct and explore loci of points satisfying a variety of interesting conditions. Use of the tools should help students develop facility in design and evaluation of procedures for construction tasks—they have limited sets of construction statements that must

be chained together to complete a drawing, and the computer produces the effects of any plan, thus giving students feedback on their efforts and the opportunity to try new ideas easily.

g. Geometry tools like fractal generators allow students to extend their study beyond the familiar family of traditional Euclidean concepts and principles to a fascinating new domain of shapes. For example, the Russian mathematician and educator Victor Minachin demonstrated two fascinating programs that allow students to conduct controlled and visually rich investigations. One showed the generation of shapes with fractal boundaries like the Mandelbrot set, revealing fundamental principles of chaos. The second helped students explore basic properties of knots by twisting, dragging, and unraveling knots on the computer screen.

h. Finally, experience with the new geometry exploration software should give students quite a new image of mathematics as an open domain in which individuals can explore with some freedom and personal control and make discoveries of their own. It seems likely that such experiences will improve student attitudes toward geometry and toward mathematics in general.

2. *Enhanced Classroom Interaction and Teaching*. Mathematics educators working with the new geometry drawing/measuring tools have also suggested that their application will cause some of the following changes in traditional patterns of classroom mathematics teaching:

   a. The teacher's role will evolve from that of an expositor, whose obligation is explaining mathematical concepts and procedures in clear and efficient ways, to that of a problem poser, guide, and partner in mathematical investigations for which students take substantial responsibility for directing.

   b. The change in teacher roles will be accompanied by an emphasis on collaborative learning in a laboratory environment. Curriculum materials will frequently take the form of laboratory assignments, launched an intriguing problem and leading to written laboratory reports. This emphasis clearly enhances student experience with, and understanding of, the experimental aspects of mathematics.

   c. As students and teachers develop the spirit of an exploratory mathematical discourse community, there will be opportunities to develop student communication skills through writing assignments and small-group or classroom presentations and discussions.

Although the preceding lists of proposed payoffs from technology use were distilled from written and oral reports related to geometry curriculum and teaching, very similar claims can be found in the literature dealing with calculators and computers in the teaching of other mathematical topics. Computer algebra systems, graphing calculators, spreadsheets, and mathematically oriented programming languages offer tools for reshaping curricula and teaching in arithmetic, algebra, trigonometry, calculus, linear algebra, and statistics as well.

Prospects

At the present time *nearly every claimed benefit of calculators and computers is a conjecture of what might result from intelligent application of technology in mathematics education*. There is still a substantial research and development agenda that must be tackled before we can confidently recommend widespread adoption of the innovative proposals. The tasks fall into three categories:

1. *Creating New Resources*. Mathematics teachers at all levels can draw on an impressive supply of hardware and software to assist instruction. However, there are still some substantial needs to create new resources. Since hardware development is strongly driven by market forces beyond the education community, it seems safe to predict that increasing computational power at more and more accessible prices is certain to continue to appear. There is similar evidence that much mathematical tool software will continue its progression to more powerful and user-friendly forms.

   Some mathematics educators will want to develop even more new mathematical tool software specifically designed for instructional use. My own impression is that, with the possible exception of tools to visualize and manipulate three-dimensional geometric shapes, we already have a supply of powerful tools that far exceed our imagination of how to use them. For me the critical

development task at all levels of mathematical education is to reexamine the goals and organization of our curricula in the light of new technologies. In almost every instance where technology has been applied to mathematics education, the first efforts have focused on doing some familiar teaching job in a more efficient or effective manner. Consideration of the more fundamental question, "Are traditional curricular goals still important?" usually comes later. Intelligent answers to that question require development and study of prototype curricula that embody new possibilities. We will then also need new assessment strategies to assess the promised effects of technology.

At ICME-7 there were some indications that people are beginning to experiment with bold initiatives in curriculum and teaching and testing connected to technology. Much of that work is taking place in North America, with parallels in England, Sweden, and Israel (see Fey & Hirsch, 1992). There are people testing the limits of change in the balance between symbol manipulation skills and conceptual understanding and problem solving in arithmetic, algebra, and calculus. But there is only modest evidence of widespread change at the school implementation level. Since the impressive geometry tool software is of more recent vintage than arithmetic calculators or spreadsheets or computer algebra systems or function graphers, developments in geometry have just begun to confront the fundamental questions about what our goals ought to be, in light of the new technology.

2. *Documenting Technology Effects.* Once innovative experimental curricula and teaching methods have been developed, it is still essential to document the effects of those new programs. There is only a modest research literature describing student learning of specific mathematical topics or general reasoning processes, patterns of classroom interaction, student use of calculator and computer tools, or teacher behavior in the new technology-rich environments.

   Going beyond simple description of technology effects, it would be very useful to have more probing studies that attempt to explain the connections among tool features, curriculum materials, classroom organization, student attributes, and student learning. Studies of this sort have only begun in a few places.

3. *Causing Educational Policy Change.* It will certainly be interesting to curriculum developers and researchers to produce new mathematics programs that exploit the power of modern technologies and to study their effects in practice. However, if those new ideas are to find a way into everyday school practice, it will be essential to collect data that compare the effects of new programs with the traditional programs they are to replace. This does not necessarily mean a "horserace" competition in which competing programs are measured on a scale of common objectives. A more reasonable approach would produce profiles of effects for new and traditional programs and encourage the essentially value-laden debate about the desirability of each. As a practical matter, to cause change in schools, this kind of comparative study must be conducted.

   Even if the new mathematics programs using technology produce attractive profiles of effects, they will not be implemented unless there is a systematic, comprehensive plan for school staff development that helps teachers prepare for their new assignments. Some ICME-7 reports suggested considerations that should be addressed in implementing such programs. The experiences of several groups working with teachers confirmed the fact that the kinds of changes being proposed cannot be easily absorbed in a short period of time.

Conclusion

As I have noted at several preceding points, much of the most exciting development work applying calculators, computers, and video to mathematics education is occurring in North America. It is likely that this research and development work will continue to produce interesting and effective curricula and teaching methods. However, if our schools are to capitalize on this creativity, it is essential that we make much greater efforts to apply those new ideas to the activity of everyday classrooms. The resulting changes will not be implemented without short-term difficulties, but I am convinced that the long-term payoff is substantial and the failure to take advantage of our current opportunities will disadvantage large numbers of students who can get so much more out of school mathematics.

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ASSESSMENT IN MATHEMATICS EDUCATION AROUND THE WORLD

Norman Webb
University of Wisconsin, Madison, Wisconsin

Assessment in mathematics education is receiving increased attention in many countries. The new initiatives in assessment accompany both curriculum reform and political restructuring. Assessment by its nature measures educational outcomes to inform students, teachers, and policy makers. Assessment bridges the world of education and the world of politics. An international discussion of mathematics education assessment requires not only an understanding of the curricula that are being implemented and the manner in which mathematics is being taught but also an understanding of government policies with regard to education, the social implications of existing curricula and government policies, and prevailing economic conditions.

During the four years since the Sixth International Congress on Mathematical Education (ICME) was held in Budapest, Hungary, a larger and more interactive group of international assessment colleagues has formed. One important reason for the strengthening of this group was a study group, formed by the International Congress, that met in Calonge, Spain, in April 1991. Mathematics educators from thirty countries attended this conference, presented papers, and produced two monographs—to be released shortly—on assessment as it is employed in different countries. This study group, together with the increased interest in assessment in many countries, enhanced the level of discussion in Quebec City and the camaraderie among those attending the assessment sessions.

Two main assessment series were offered at the seventh ICME. One four-session series focused on the assessment study that grew out of the Calonge conference and treated other topics that were addressed at the Calonge meeting. Eleven presenters reported on issues and programs that were raised previously at the Calonge conference. The series at Quebec that treated these matters began with an overview of the main themes and findings of the Calonge study group; the series then considered, in turn, critical and analytical aspects of assessment, cases of innovative and experimental assessments, and discussions of technical issues in assessment. The study sessions were well attended, since there were more than 100 participants at each session. However, the size and format of the sessions limited discussions and interaction among the participants and speakers.

A second assessment series was designed to generate discussion. Working Group 8, one of sixteen designated working groups, focused on the theme of assessment. The four 90-minute morning sessions in this series treated five themes: the paradox of assessment in mathematics, assessment policy, assessment of attitudes and dispositions, assessment of problem-solving abilities, and diagnostic assessment. The first session set the tone for the working group. The second and third sessions convened in five smaller groups—one for each area. These sessions began with the presentation of papers by one or two speakers and ended with open discussion. The fourth and final session consisted of reports from each of the five subgroups. More than 130 participants from twenty-four countries attended the assessment working group. Juliana Radnai-Szendrei, one of the organizers of this working group, facilitated communication within the group by preparing and distributing a premeeting document that contained the papers of most presenters. In addition to the two main assessment series, a few other hour-long presentations treated such assessment topics as Ontario's Graded Assessment in Mathematics, the Third International Mathematics and Science Study, and the Second International Assessment of Educational Progress.

Several major themes were interwoven into the more than fourteen hours of assessment presentations. I have chosen two themes to help organize my impressions and reflections on the meeting. One theme concerns change in assessment practices. In many countries, assessment is in a state of transition, and new ways of thinking about assessment are emerging. I will attempt to summarize some of these changes in an effort to clarify the "state of mathematical assessment in the world." A second theme centers on the nature of assessment. During the Quebec congress, many of the questions that were posed by presenters and discussants dealt with this issue. Participants asked: What is assessment? How is assessment viewed differently by different members of the educational system—administration, teachers, students, and the public? What are the social functions of assessment for a society? How is assessment related to other

components of the educational system, such as the curriculum, instruction, teachers' roles, and policy? When posing these questions, participants clearly attached different meanings to the term *assessment*. The cross-cultural perspective on assessment that we obtain from participants at the congress provides a broader view of assessment than we obtain when we analyze the role of assessment in only one educational system. Although none of the presentations addressed technical issues in great detail, these issues were treated indirectly in many of the talks and were noted explicitly in a number of presentations.

Changes in the Practice of Assessing Mathematical Knowledge

Assessment of what students know about mathematics and of what students can do with mathematics is undergoing change in a number of countries. In some countries change is the result of a national initiative. In other countries, the impulse for change derives from action at regional, state, or provincial levels of organization. In still other countries change is directed toward specific innovations. Claudi Alsini (SPN) poignantly characterized the wide range in the conditions of education around the world, and thus the variance in attention that can be given to assessment, when he commented, "To have a nonsmoking airline requires having an airline." Although many countries are striving for innovation and change, what can be done is largely determined by current situations and conditions.

Changes in England are related to the reform of the national curriculum. Ten attainment levels have been defined for a national assessment that has been imposed by law. Schools are legally required to publish their assessment results for pupils at ages 7, 11, 14, and 16 by reporting the distribution of students at each attainment level. Standard national tests for ages 7, 11, and 14 are organized through the School Examinations and Assessment Council. Terminal examinations for age 16 students are organized by five private examination groups. In the initial proposals for the new assessment system the teacher assessment component was more significant than external examinations. However, after a period in which there were many changes in the secretaries of state for education, teacher assessment can now constitute no more than 20 percent of the marks for assessment of 16-year-olds (Brown, 1992).

Assessment instruments in England have changed by including more problems that are situated in a context, by administering different papers to different pupils—based on the pupils' attainment—and by using more comprehensive tasks, such as reading an article and responding to questions based on the article. Although open-ended tasks, investigations, problems based on production situations, and other alternative forms of assessment have been developed by different projects, schools in England still differ greatly in their implementation of these new procedures. One of the central issues that underlies efforts to bring about change in assessment concerns the willingness of educational systems to reduce reliability and consistency to increase the implementation of more interesting mathematical experiences.

Curriculum and assessment reform has been under development in the Netherlands for more than ten years. A greater emphasis is being placed on "mathematizing," studying mathematics in relation to its real-world applications. The national examinations that are given to those who complete their secondary schooling after five years of such instruction require that students solve five big problems, with approximately twenty questions, over three hours. The problems are derived from applications of mathematics, such as the regulation of flow of traffic by traffic signals at an intersection, or the operations of a fish farm. Pupils' examinations are graded by their own teachers, and a sample is scored by another teacher.

Australia has six states and two territories, each with its own educational system. Supported by an endorsement of the Australian Education Council, some state and local universities are striving to broaden the conception of mathematics education for all young people. Across the country, prototypes of assessment tasks are emerging for use in primary and secondary schools, including multiple-choice questions in simple and enhanced formats, questions that require explicit responses, open-ended questions, extended questions, and open-ended investigations. One state, Victoria, requires that the significant number of year 12 students who are taking mathematics in their final year of secondary school complete four externally designed instruments, called Common Assessment Tasks (CATs). CAT1 is an investigative project on a centrally set theme that is worked by the students over fifteen to twenty hours. CAT2 is one of four centrally set problems that is worked over two weeks, half of which time is to be spent in class. CAT3 is a 90-minute multiple-choice test on concepts and skills. CAT4 is a 90-minute test on approximately four structural questions that are graduated from routine to nonroutine. Teachers grade their own pupils on these different forms of assessment—assigning grades that have to be defensible to their pupils and to teachers in

other schools. Additionally, teachers in a given school are required to confer with each other about their application of the assessment criteria. Subsequently, they are to share their assessment criteria and results with teachers in other schools. It is important to note, however, that what is being done in mathematics assessment in Victoria cannot be taken as representative of all of Australia. An entirely different approach to assessment has been adopted in Queensland, where no external examinations have been used for twenty years, and where mathematics assessment is school-based and more dependent on teachers. There are activists in Australia who are working toward changes in mathematics assessment at both the classroom level and external assessment. These activists are striving to educate teachers on ways that assessment can be more closely aligned with the curricular goals of having students perform mathematical inquiry, apply mathematics, appreciate mathematics, and have a positive attitude toward mathematics.

A number of the countries represented at ICME were not undergoing national changes in assessment but had people and projects investigating new forms of assessment. Leonor Cunha Leal and Paulo Abrantes (POR) reported on an approach to assessment that is being developed for an innovative curriculum project for grades 7–9. They are using two-stage tests (assign homework, then give feedback and have students further develop their work), project work, and oral presentations to advance the curriculum goals of having students understand the role and relevance of mathematics, creative a positive attitude toward mathematics, and solve problems mathematically. Ching-Lin Hu (CHN) reported that the prevalent form of mathematics assessment was the "single written assessment" given students at the culmination of an educational experience, either at the end of a unit, semester, schooling, or before entry into a higher school. In Ching-Lin Hu's province, teachers are using a two-part model to assess the "mathematical potentialities" of middle school students. Grades are given on mathematical capacities, based on students' ability to perform abstract generalizations, "nimble" selection and application of mathematical knowledge, and scientific exposition and reply. A second grade is given on a student's disposition, based on refinement and quality of thinking, willpower for surmounting difficulties, and a realistic and meticulous style of doing mathematics.

John Clark (CAN), of the Toronto Board of Education, gave an hour presentation on his school district's Benchmark Program. Over five years, benchmarks have been developed to represent standards of student achievement in mathematics at the end of grades 3, 6, and 8. The district has recorded actual interviews of students while they performed assessment tasks, and it has prepared videotapes along with holistic scoring of the students' performance, and paper-and-pencil work. Teachers in this district, which serves 72 000 regular day-school students, are then given access to the benchmark libraries at one of the three grade levels, so that the teachers can design their own means to assess student learning and evaluate their classroom programs. The district has developed a long-range plan that includes extensive in-service programs and strategies for teachers to work cooperatively. Toronto's efforts to reform assessment are based on the assumption that—by providing teachers with information about student achievement in activities that teachers use in their classrooms—teachers will make more informed judgments about the quality of their students' work. Central to this system are the benchmarks, or models, that can be used as reference points (Clark, 1992).

Anne Arundel County, Maryland, has a district that is comparable in size to that in Toronto. The district accepts the proposition that "what gets measured gets taught." Assessment reform in the district is influenced by new state assessment programs in mathematics that are implemented annually to evaluate students in grades 3, 5, 8, and 11. Each grade level is divided into four instructional blocks of nine weeks. A county-wide mandatory assessment is administered at the end of each instructional unit, and the results are used to evaluate students. The assessment is divided into two parts: one part is traditional, and the other part consists of performance tasks, in which students must answer questions that are based on one situation. The questions require students to use different forms of representations and draw conclusions based on information that is generated in other questions.

One of the most important inferences that one can draw from the seventh ICME is that change in assessing mathematical knowledge is being considered in many countries. Participants reported developing or implementing assessments of mathematical knowledge that are situated in contexts that call for the application of mathematics, the use of more complex tasks requiring the integration of mathematical knowledge, and the requirement that students perform tasks over extended time periods.

Nature of the Assessment of Mathematical Knowledge

A second major theme among participants at the seventh ICME was revealed in discussions and descriptions of the nature of assessment and the manner in which assessment is applied in mathematics education. Antoine Bodin (FRA) described assessment as the "means to organize (or to study) some situations in such a way that it enables the gathering of information which after processing can reveal something that is reliable about personal (or collective) knowledge." Bodin went on to note that assessment can be something that is imposed from the outside on a didactical relationship that disturbs the phenomena that are to be observed. Any one assessment is incomplete, in that two students who have done the exact same work on one assessment can look different when one additional bit of information about their performance is added to the assessment. Tom Romberg, of the University of Wisconsin, stressed the need to analyze critically what is done in assessment because assessment is highly inferential. In his words, "We can never really capture what a person knows about a subject. We can only make inferences from data."

Discussing assessment in an international community highlights the political and social ramifications of assessment. Jim Ridgeway (UK) reminded people that the implications of mathematics education assessment cannot be ignored. He provided data that demonstrated that assessment is used as a screening process to exclude students from higher levels of education, to enhance or reduce the social status of students in some cultures, to explain away social inequities, and to inhibit the concept of learning for its own sake. Great care needs to be taken to ensure that the nature of mathematics and the learning of mathematics dominate change rather than assessment. In colloquial terms, "You don't make the pig fatter by weighing it repeatedly." More particularly, Tom Romberg reported that data from a national survey of grade 8 mathematics teachers show how large-scale, external assessments influence a high percentage of teachers.

Psychological theories interact with assessment practices. Kevin Collis (AUS) described the development of assessment instruments that are based on a psychological model. He and other researchers have found that high school students have three modes of intellectual functioning—sensorimotor, iconic, and concrete symbolic—that they can bring to bear when they solve novel mathematical problems. Other participants reflected on assessment in relation to the popularity of constructivism. If the premise is that knowledge is actively constructed by individuals, then what are the implications for assessment? The situation becomes more complex if knowledge is constructed in a social context. Ole Bjorkqvist (FIN) reminded participants that if knowledge is constructed by the learner, then large-scale assessment that gives common tasks to a large number of students will fall short of revealing what individual students know. He argued that, at best, assessments can give only a very general picture of each student's knowledge of mathematics.

Assessment is one component of the educational system. The products of assessment are not the ends of mathematics education but are data that can be used to make inferences and decisions about students and programs. In this regard, Steig Mullen-Olseg (NOR) stressed the importance of keeping education student-centered. He drew attention to the need to consider whether the goals guiding assessment are the goals to which students have agreed. His comments relate to practical hermeneutics, the notion that learning begins with students as part of the process.

Teachers perform a role in assessment that is sometimes ignored. Significantly, it was generally the teacher participants who observed that while reform in assessment is important, the demands that are placed on teachers are frequently neglected. In addition to managing new forms of assessments, change in assessment also challenges teachers' knowledge of mathematics and their understanding of how students learn. Tom Schroeder (CAN) reported on a study that illustrated the point that teachers' expectations about the approaches that students take in performing an assessment task do not always coincide with the manner in which students actually approach the task. Teachers also may vary in their assessment styles, inasmuch as some teachers are more effective in using one approach than they are in using another. Further discussion of the relationship between teachers and assessment suggested that students' assessment results can be influenced by teachers' insecurity. Rather than measuring students' knowledge, assessment may be measuring a teacher's ability.

Hugh Burkhardt (UK) identified two roles for assessment: performance measurement and curriculum influence. Although there are other views of assessment, for example, that it is externally imposed or teacher-driven, the other views are related to these two purposes. Assessment is used in different countries and by different people for a variety of ends. The multiple purposes of assessment require clarification of the

situation in which assessment takes place and the manner in which assessment interacts with the curriculum, teachers, and students. This clarification must be accomplished if one is to understand assessment. For some, assessment leads reform and can be used as a lever for change. For others, assessment is an integral part of the system, and changes in assessment should accompany changes in curriculum and instruction.

The multiple functions of assessment increase the technical issues. People from a number of countries noted that they are attempting to ensure that assessment be more representative of goals in mathematics education that are directed toward the development of student skills that enable them to apply their knowledge, work on extended tasks, and appreciate mathematics. But in increasing the validity of the assessment to reflect these goals, reliability can be lowered, costs increased, development complicated, and corruption encouraged. Although participants raised technical questions that bear on these issues, little was reported about efforts that would address and resolve these issues.

Summary

In reporting on the Quebec City meeting of ICME, I have tried to convey some of the flavor of discussions concerning assessment. I have not reported on some of the sessions that were more descriptive than issue oriented. For example, two sessions were held on the design and results of the international assessment studies. The discussions on assessment at the conference were interesting and raised important issues. Internationally, assessment is receiving a considerable amount of attention. Mogen Neiss (DEN), the co-chair of the ICME study on assessment, summarized that study's findings. He noted four points: concepts in assessment need to be clarified; assessment plays too important a role to be neglected; resolving the mismatch between goals and practice is a challenge; and suitable modes of assessment already exist but cannot be reconciled with current conditions or traditional psychometrics. These points and discussion on assessment in Quebec City only confirmed that assessment in mathematics education has many issues to address and challenges to surmount as we look forward to the eighth ICME in Seville.

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POSTER SESSIONS: A REFLECTION OF WORK IN PROGRESS

Bonnie Shulman
Department of Mathematics, Bates College, Lewiston, Maine

I came away from ICME-7 feeling as if I had attended an international potluck and eaten too much. But the offerings were all so scrumptious, I had to sample as many as possible. Writing down my impressions helps me to digest and assimilate this feast. My particular tastes led me to sessions on calculus reform in the undergraduate curriculum and the uses of writing in teaching mathematics. I teach at a small liberal arts college in Maine. Beginning in the fall of 1992, we will teach all sections of our calculus sequence using the Harvard Consortium materials and graphing calculators. I also teach a first-year writing seminar called "Calculus for the Queasy." I use the history of calculus to teach writing (the inverse problem to using writing to teach mathematics). For me, the most stimulating and instructive components of ICME-7 were the poster sessions.

L. Diane Miller (USA) examined how teachers benefit by reading their students' writing assignments. They gain insight into their students' conceptual understanding and can better diagnose individual difficulties. The bond between teacher and student is also strengthened by this additional mode of communication. Arlo Schurle (USA) candidly reported on student attitudes to writing in mathematics classes and his own frank assessment of the benefits for their learning. The majority of students feel that writing assignments help their learning, but Schurle admits their actual performance on examinations did not improve. It occurs to me that if there had been some essay questions on the examinations as well, students might have been able to exhibit their perceived gain in mastery of the concepts. Schurle also points out that the grading of writing assignments can be a staggering drain on a teacher's time. He provided some useful examples of short assignments that required minimal time to grade. Some forms of writing (e.g., journal writing) often work best if one reads and responds to them but does not grade them with a letter grade. Janet Barnett (USA) displayed some examples of weekly writing assignments and student responses that were very enlightening, and I will (with her generous permission) be able to freely adapt and use her ideas.

Also inspiring for their concrete suggestions were the posters of Dorothy Buerk (USA) and Janet Martin (USA). Buerk, at Ithaca College, asked her students to write metaphors for mathematics. She posted some examples, and they were powerful, exciting, and often distressing. As she pointed out, the insights that such assignments provide to our students are invaluable, because their perceptions of mathematics strongly affect their performance in the classroom. One will teach differently to the students who view mathematics as "quicksand" or an "abusive parent" and those who perceive it as similar to "designing a quilt" or "negotiating a maze." Martin asked preservice elementary teachers to write mathematical autobiographies in order to understand the nature of their mathematical fears and anxieties. Their responses were fascinating and, again, often disturbing and had implications for the reform of mathematics education at all levels. High school and college mathematics teachers clearly create a great deal of fear and anxiety for many preservice teachers, which they then pass on to the next generation. I caught a glimpse of this process in my writing class, where students told me mathematics was often used as a punishment ("If you don't behave, we'll do math!"), something like dropping down for a hundred push-ups in the army.

Emilie Kenney (USA) of Siena College, with William Rosenthal (USA) of Ursinus College, expertly made the important point in their poster that the teaching of mathematics is not purely objective but rather occurs within social and political contexts. They illustrated this by juxtaposing standard problems from calculus texts and proposing alternative problems with identical mathematical content. It became obvious to me that our choice of problems can serve to undermine or support the dominant point of view or paradigm of a given culture, in this case the militarism and capitalism of Western industrialized nations. Kenney's alternative problems were provocative and stimulated me to make up some of my own.

My mind is still spinning from this rich smorgasbord of ideas. Luckily, I have the upcoming school year(s) to put them into practice as I continue both teaching and learning from my students.

JUST ONE MORE COMPUTER, PLEASE

Jacqueline Mayo
South Carolina State Department of Education, Columbia, South Carolina

Have you ever felt frustrated with an overcrowded classroom of thirty-five to forty students? Do you find yourself begrudgingly having to pay for supplies occasionally? Have you ever wondered if there would ever come a time when your classroom would have a sufficient number of manipulatives, calculators, and computers so that adjustments would not have to be made in your instructional program? Most educators would say that, at one time or another, they have been confronted with a shortage or scarcity of resources. With this mindset, I attended ICME-7 and participated in the working group that addressed scarcity of resources for mathematics educators. I did not realize at the time that this group of people would forever change my perceptions and the way that I would view the resources that are available to mathematics educators in the United States.

At the onset of our discussions, all of us were reminded of the statements that were made at the opening session of ICME-7: one-fifth of the world's population owns 80 percent of the world's resources, and one-fifth of the population owns less than 1.5 percent of the world's resources. At this point, all participants were asked to share their views concerning how scarcity of resources has affected the mathematics instruction in their respective countries.

A delegate from Nigeria related that in many primary classes, it is not uncommon to have 150 students per teacher. In addition to the large student-teacher ratio, there is a shortage of instructional materials. This has caused some teachers to feel helpless.

The delegate from the Dominican Republic stated that many teachers have only blackboards and no textbooks available to them. The teachers have to use their own funds to buy chalk. Because of the low pay that many of these teachers receive, certified teachers are leaving the classroom and going into industry. Over the past decade, the percent of certified teachers has decreased from 70 to 50, replaced often by high school graduates.

In Bhutan, a country that borders on India and China, many children sit on the ground to be taught because there are no chairs or desks. Books and slates are a rare commodity. Children must be taught mathematics by using their bodies, sticks, and stones. When place value is taught, the students use their arms to stand for numbers. When operations are taught, the students themselves become the numbers. Because of the high rate of malnutrition in this country, having parents knowledgeable about mathematics is imperative. They need to be able to record their child's weight daily on a graph. They should also be able to count the number of breaths per minute, which could help them to predict a case of pneumonia. Unfortunately, only one in four children attend school, and only one in sixteen complete the sixth grade.

Research has substantiated the fact that mathematics achievement is directly related to material resources. Thus, countries that have insufficient resources are at a severe disadvantage not only in regard to the education that is available for the masses but also in regard to improving the economy of their country. The old adage that mathematics is the language with which that God wrote the universe is certainly true. Without a sufficient command of this language people have limited access to the high technological society that we live in today.

Since humanity operates on an international scale rather than a national one, the following questions must be viewed from that standpoint. Does the United States actually have a scarcity of resources? Does not having a sufficient number of manipulatives, calculators, and computers constitute a shortage of resources? If one believes that there is not a shortage of resources in the United States, then perhaps a few fundamental ethical questions may be at stake: Should educators feel an obligation to share their resources with those countries that have fewer? Should these resources include collaborative human efforts that would respect and preserve these countries' cultural differences? The discussion begins now!

THE IMPACT OF THE CALCULATOR ON THE ELEMENTARY SCHOOL CURRICULUM: REFLECTIONS ON ICME-7

Betty K. Lichtenberg
University of South Florida, Tampa, Florida

My main professional focus at ICME-7 was on the working group dealing with the impact of the calculator on the elementary school curriculum. There are some exciting projects all over the world devoted to the effective use of calculators in mathematics instruction. Twenty-three papers representing ten different countries were presented in the working group. It is only possible to comment on a few here.

The Calculator-Aware Number curriculum is used in England and Wales. The project, directed by Hilary Shuard (UK), started with six-year-old children in 1985 and is continuing. Each child has a calculator always available. Standard algorithms for the four operations are not taught, and children are encouraged to become proficient at mental calculation. The children devised some ingenious, nonstandard algorithms and developed knowledge of large numbers, decimals, and negative numbers much earlier than in the traditional curriculum. Evaluations of Calculator-Aware Number students have revealed highly favorable results on the regular examinations, and the attitudes of the teachers involved are quite positive.

In 1989, England and Wales introduced the National Mathematics Curriculum, and this was somewhat influenced by the Calculator-Aware Number project. Although the National Curriculum does require all students to use calculators, it still retains noncalculator methods. It seems to represent a "half-way stage" in the modernization of calculation in the primary school. Some United Kingdom speakers were suggesting that they were helping teachers to "subvert the National Curriculum!"

Australia has long been a leader in promoting effective use of calculators at the primary level. Susie Groves (AUS), one of the team leaders of the Calculators in Primary Mathematics project, spoke about their goals, which include changing attitudes, expectations, and practices of the students and teachers. They have developed a network of teachers and a newsletter for communication, unusual at this level of education. A fruitful idea that teachers are using frequently is SCAT (Some Calculator Activity Time), to be done in spare moments by choice. This is analogous to, and as enthusiastically endorsed as their well-known SQUIRT reading time.

In the United States, the Calculators and Mathematics Project–Los Angeles (CAMP-LA), directed by David Pagni (USA), focuses on integrated activities in grades K–6. Some of the major goals of this project are to emphasize conceptual development, numerical relationships, and applications in real-life experiences; to introduce more complex mathematical problems at an earlier age; and to remove computational constraints so that students can focus on the processes of solving problems and develop problem-solving skills and strategies.

A model for integrating concept formation in mathematics with calculators and other resources was presented by Donna Berlin (USA) from Ohio State University. Her project focuses on place value. The approaches used view the topic through concrete, semiconcrete, and abstract representations combined with static, animated, and manipulated movements in the instructional model. Berlin has also compiled an ERIC Calculator Search and Summary for Working Group 16, which will be available in the near future.

Common threads among the papers in this working group include encouraging mathematical thinking, exploration, mental work, having fun with mathematics, and posing questions like "What do you think will happen if ...?" The most widely held belief of the presenters was the importance of integration of the calculator and appropriate mathematical content.

Reflections on ICME-7 (or why you should go next time): Collegiality is unsurpassed at these meetings. There is time and opportunity to renew old acquaintances and to make new and lasting friendships. The hospitality and facilities of the host university were excellent. We were treated like visiting dignitaries. The weather was beautiful, and the city and its food and ambiance were unparalleled. Quebec City is one of the prettiest cities in the world. A conference cruise on the St. Lawrence provided a beautiful view as well as information on the historical and geographical features of the city. The U.S. Commission

on Mathematical Instruction dinner meeting was professionally enlightening and enjoyable, too. All these things combined to make ICME-7 memorable.

Adieux, mes amis. Au revoir. Nos vemos en Sevilla en 1996!

IMPRESSIONS OF A SECONDARY SCHOOL TEACHER AT ICME-7

Arnold Perham
St. Viator High School, Chicago, Illinois

In this short presentation, it will be impossible to review all the related topics presented and discussed at the conference. However, I would like to highlight a few items of interest to secondary school teachers.

Colette LaBorde (FRA) and Heinz Schumann (GER), lecturers, spoke about the renewed interest in the inductive method for concept formation in geometry as a byproduct of computer graphics. Although both of them demonstrated their point by using *Cabri-géomètre*, developed at the University of Grenoble, other software, for example, the Geometer's Sketchpad and the Supposer, effect similar results using different structures.

Students in traditional geometry courses are limited to drawing static diagrams. Graphing software, however, changes the screen into a dynamic field. Consider this example: a student, using menu and mouse, plots three points on a screen. Effortlessly, the points are connected with line segments and the perpendicular bisector of each segment is displayed, and they, of course, intersect at the circumcenter.

At this point the student begins to experiment, producing dynamic rearrangements of the original plot. As the student "clicks on" to a vertex and "drags" it across the screen, the perpendicular bisectors simultaneously reorient themselves as the circumcenter takes up positions in, on, or outside the triangle. As the vertices approach collinearity as a limit, the perpendicular bisectors become more parallel, and the circumcenter approaches infinity.

Geometry graphing software raised a number of significant pedagogical considerations at the Congress in both formal and informal discussions. These exchanges touched on dynamic graphing and the opportunities and problems it presents to teachers in guiding the learning process. Inevitably, experimenting with images leads a student to conjectures and generalizations (theorems). Since these images are the product of convincing graphics, the student might not see the necessity of a traditional geometric proof. Heinz Schumann does not consider this to be a major problem, since students usually understand that graphics indicate what happens but not "why" it happens. To answer the "why" requires reflection beyond the images.

A more serious problem for the teacher lies in the fact that a student's generalization from graphics experimentation frequently might not fit into the topic currently under discussion. Graphics experimentation would place demands on the teacher to guide the learning process, incorporate student discoveries, follow the syllabus, and value students' newly found discoveries.

Another suite of software, Maxima, presented by its creator, Douglas Butler (UK), is another example of ever more flexible, powerful, faster algebraic graphing software. In using this software, a student enters the equation the way it is written on paper:

$$(x - 2)^2 - 3(y - 2)^2 = 9$$

As teachers know, software is not generally transportable from one system to another as with audiotapes. Since the above-mentioned software was created using BBC BASIC for the ACORN computer, it is not currently available for typical systems used in the United States. Perhaps the teachers' hoped-for day will come when the software is generally interchangeable among different computer systems.

Although computers are generally more flexible and powerful than graphing calculators, Hannah Perl (ISR), in her presentation, highlighted an important consideration for mathematics teachers, namely, computers are not immediately available for mathematics students as they do their classroom work. As a result, she feels that the graphing calculator is a more important pedagogical tool than the computer on a day-by-day basis.

In the final lecture of the conference, Benoit Mandelbrot (USA) underscored the importance of graphics images for a basic understanding of geometry. He said that every age uses the tools available to itself. The produced images are necessary for the "eye to guide the mind."

ICME-7, QUEBEC, CANADA

Richard Sander
Ketchikan High School, Ketchikan, Alaska

This was my first ICME; in fact, this was my first exposure to mathematical conferences other than NCTM national conventions. It took at least a day to get used to the "lecture" format of most presentations. As a high school mathematics teacher, I have been dispensing with "full frontal" teaching and replacing it with a "guide from the side" style. What an awakening I got at this conference!

I chose Topic Group 8, "Teaching Mathematics through Project Work," since I am interested in how this approach would work in a typical high school setting. Led by Jarkko Leino (FIN), both days consisted of four short presentations, followed by a very short period for questions.

The initial premise given was that in order for a student to recall facts, the information must be broken down into small, fragmented pieces. Typically, these facts are unrelated to the student or the student's past experiences and, therefore, lose meaning in the process. In the constructivist view of learning, which capitalizes on students' prior experiences to allow them to construct meaning from new experiences, projects are in order.

The following reasons for doing projects were mentioned by all the speakers: mathematical learning in context, active learning (not a spectator activity), projects help motivate students, problem solving is central (the main reason we study mathematics in school), cooperative atmosphere, and benefits of student presentations as assessment. Some expected outcomes expressed by John Berry (UK) included skills developed through problem solving such as forming models, collecting data, and testing models.

In general, all the discussion leaders presented their *own* work and agreed that project work could be used as a vehicle to encourage skill learning. In my secondary school, we have been teaching content in the context of actual problems, and I was very comfortable with this concept but felt that direction toward assessment was lacking.

In Working Group 13, "The Place of Algebra in the Curriculum," we looked at a historical perspective, the present situation and the future. Some questions left unanswered were these:

- Are learning difficulties local or global in nature?
- How do we combine meaningful activities with manipulative learning?
- What is the role of graphical representation in secondary school algebra?
- How does the ready availability of symbolic manipulators affect our typical algebra curriculum?
- Do we need to differentiate carefully between evidence and proof (in light of current technology)?

As we discussed algebra, it became evident that technology not only can give insight into the topic of algebra but also bring forth new problems, new concerns, and new directions.

Bert Waits (USA) demonstrated the use of the TI-85 hand computer to solve some problems (variations of maximizing the volume of a box) usually associated with calculus. It appeared to me that he was actually advocating a very conservative notion of the future by solving the same kind of problem we can do without technology.

Having used this new tool (TI-81) in high school classrooms for the last year, I firmly believe that this technology helps present algebraic material to a wider audience. Students have ready exposure to the connections between an algebraic and geometric representation of a problem. The real question for me is, "When and how to make use of 'black box' technology and symbolic manipulators for 'doing' algebra?" In his lecture, Seymour Papert (USA) posed a question that has helped me with my dilemma. He simply asked, "*What will be the place of reading in 5–20 years, as a means to seek knowledge?*"

I left Quebec with many unanswered questions—I wouldn't have it any other way.

COMPUTERS, MATHEMATICS, AND LEARNING: NOTES FROM ICME-7

Paul Zorn
St. Olaf College, Northfield, Minnesota

How have computers changed mathematics—how it's done, how it's taught, and how it's learned? What changes can we expect in the future? The theme of mathematical computing, not surprisingly, surfaced in almost every lecture, working group, and discussion session at ICME-7. Here are some samples of questions raised and answers suggested:

- *How much* computing technology should students be permitted—or required—to use in studying mathematics at various levels? The question raises thorny economic, social, pedagogical, and disciplinary questions. How, for instance, does omnipresent technology change what it means to know and do mathematics? Deep pedagogical changes are needed, said David Tall (UK); we will need to teach mathematics not just to students, but to students-with-machines.
- How ingrained has computing *already* become in mathematics education? Bernard Cornu (FRA), summarizing results of a multiyear ICMI (International Committee on Mathematical Instruction) study, observed a surprise. Although mathematical computing has *strongly* influenced the reputedly conservative mathematical *research* establishment, it has seen *less* use than expected in mathematics *education*. In his plenary lecture, Seymour Papert (USA), who developed Logo, suggested a reason for computing's relatively slow advance in schools. According to Papert, computing challenges many of the basic assumptions and principles that define "school," including conventional divisions among subject areas and ready measurability of results. Faced with such challenges, Papert observes, "school" behaves like a threatened *organism*, "fighting back" either by rejecting technology altogether or by co-opting technology, cunningly, to minimize its threats.
- Many students see mathematics as an austere, gray, ethereal subject, remote from human experience or intuition. Computers can bring mathematics back to earth, said various speakers, re-connecting it to visual and physical experience and intuition. Benoit Mandelbrot (USA) stressed (and demonstrated) mathematics' "green and golden" visual side, fully visible only with the twin aids of high-powered computing and a trained eye. Jim Kaput (USA) argued for grounding students' mathematical knowledge in their own physical or other experience. The crucial idea of elementary calculus is to describe phenomena of *change* mathematically. Kaput demonstrated one way computers can help: by simulating—but also mathematizing—a driver's experience on the highway, technology helps ground ideas of amount and rate in daily experience. (Such "physical" demonstrations, as Kaput and others observed, nicely illustrate the primacy of *derivatives* in mathematical descriptions of nature: drivers' and pilots' direct, gut-level experience is with velocity and acceleration, *not* with position.)
- Mathematics education conventionally proceeds from the "simple" (e.g., polynomials) to the "complex" (e.g., transcendental functions). Technology may alter this familiar order of precedence as our ideas of what is simple and what is complex change. Polynomials, for instance, may be "simplest" from the viewpoint of paper-and-pencil manipulation, but exponential functions may be more natural, and thus "simpler," for modeling physical phenomena.
- The relationships between *discrete* and *continuous* phenomena pose difficult "epistemological obstacles" for students in elementary analysis.

Computers, in a sense, embody discrete points of view—even when drawing apparently continuous graphs, for example. A deep understanding of mathematics requires that *both* points of view be understood. Computers may help draw out, and perhaps alleviate, students' mathematical difficulties with these subtle issues.

The vexing discrete/continuous dichotomy even poses *historical* questions. For example: Why did the Greeks first grapple with questions of continuity, whereas earlier civilizations were content with discrete phenomena? Perhaps, suggested one speaker, because of different civilizations' different ideas of divinity.

The Greeks recognized a virtual continuum of deities (gods, demigods, demidemigods, . . . , mortals) whereas the Hebrews recognized only a God/mortals duality. (Whether computers can help resolve such *theological* obstacles is left to the reader's imagination.)

- Do computers—computer graphics, especially—mislead students mathematically? Yes, said several speakers; for example, computer graphics can blur the important distinction between a discrete point set and a continuous graph. Not likely, said Benoit Mandelbrot—fear of mathematical pictures is an "old canard." Pictures mislead only when they're *intended* to do so; computers, without axes to grind, draw "neutral" pictures.
- Technology may permit—or even oblige—us to resequence the standard liturgy of mathematical topics in education. As David Tall remarked; computing can *reverse* the usual progression from theorem to proof to computational applications. With computers to simplify and speed calculation, students can approach mathematics as Euler did: "calculate, calculate, calculate"—*then* prove things, sometimes.

Seymour Papert made a related prediction, applied to *all* areas of education. When technology revolutionizes how information is organized and accessed (even by preliterate children), students simply "won't sit through" anything like the present school fare. It's important, Papert advised, to begin preparing now for the near future's radically different learning environments.

- How will technology change the mathematical content students encounter? What new topics or points of view will emerge, or find new emphasis? Which old ones will disappear?

Various speakers suggested various answers. Seymour Papert predicted that computers will boost approximate and iterative points of view, even in early education. Benoit Mandelbrot (and many others) championed more experimental, investigative, experience-driven approaches to mathematics; some speakers saw the traditional walls separating pure and applied mathematics crumbling, or even tumbling down.

Zalman Usiskin (USA) observed that while numerical *arithmetic* pervades popular culture, *algebra* is still popularly seen as an arcane, even mystical, pursuit. (Usiskin illustrated his thesis by finding scores of arithmetical references on the front page of a local newspaper. By contrast, Usiskin found only one algebraic analysis—of Michael Jordan's jump—in a copy of the *Chicago Tribune*. Even then, the paper got it wrong. A (correct) algebraic derivation of Jordan's *parabolic* path was accompanied by a graphic of the basketball star soaring, miraculously, through a *semicircular* trajectory.) But there is hope, said Usiskin: when inexpensive, portable technology handles algebra as readily as it now does arithmetic, algebra may follow arithmetic into popular discourse.

Innumerable speakers made the general point that as computers do more and more manipulations (symbolic and graphic, as well as numeric), students will require a more conceptual, flexible, "top-down" understanding of their mathematics, one that goes well beyond a facility for performing tightly prescribed paper-and-pencil tasks. Developing such an understanding—and the roles computers might play in doing so—poses today's $64,000 questions for mathematical education.

LESS THEORY, MORE ACTION

Florence E. Klimas
Branchburg Central School, Branchburg Township, New Jersey

Plenary sessions, lectures, working groups, and topic group presentations were varied and informative. I participated in Working Group 11. This group's presentations centered on the role of geometry in general education. An example is the presentation by Susan E. Williams, University of Houston (USA), on "Spatial Learning in Video Game Playing." Nintendo's TETRIS software and a seven-year-old boy were the center of her research. A member of the audience suggested the use of Radio Shack's STACKABLES, which would be a less expensive addition to a classroom and would provide similar spatial learning experiences. Continuing in the theme "Geometry in Informal Settings," my presentation of "The Geometry of the Quilt Block" combined paper-folding exercises for the development of the geometric forms used in quilt-block designs with the changes in designs produced by the use of color. Examples of my seventh-grade students' quilt-block designs were on display in the meeting room. Each of the three sessions of this particular group had a stimulating question-and-answer period, with the section's coordinators having difficulty in closing the meeting in the prescribed time.

Learning experiences moved quickly from working groups to lecture rooms to the poster presentations to the informal origami lessons to the casual lunchroom gatherings to walks together from one session to another. It was difficult to find time to attend the exhibits and poster presentations. The Shell Centre's, which covered approximately one-third of the exhibit floor, was very informative. I attended three of their half-hour presentations and now have new ideas, ranging from designing open-ended questions and alternative forms of assessment to student-made pop-up cards.

Each day I found myself gravitating back to the more than 350 poster exhibits to talk to their authors. The posters were visited throughout the day, not just during the time periods when the authors were scheduled to discuss their presentations. I was pleased to discover that the entire supply of 275 copies of my poster (translated from English to French, German, and Italian) was exhausted by Saturday morning. Also, a supply of precut circles was depleted by those experimenting with the directions for producing models of cones, tetrahedrons, and truncated tetrahedrons described on my poster.

ADDRESSING GENDER DIFFERENCES IN MATHEMATICS

Denisse R. Thompson
University of South Florida, Tampa, Florida

The study group sessions of the International Organization of Women and Mathematics Education (IOWME) focused on theoretical and cross-cultural aspects of addressing gender differences in mathematics. The four study group sessions included two panel discussions and several small-group discussions designed to develop the perspectives raised during the presentations by panel members. This paper summarizes the panel discussions.

Four theoretical models for addressing gender differences were raised during the first panel discussion: an intervention perspective, a segregation perspective, a discipline perspective, and a feminist perspective. The intervention perspective was addressed by Teresa-Anne Mashego (SA) and made reference to mathematics anxiety. Mathematics anxiety is a learned construct that affects females more frequently than it affects males. Mashego discussed the idea that situations involving trauma often lead to fear and anxiety. While these are normal reactions, when trauma is combined with a mathematics situation resulting in fear and anxiety, a pattern of conditioning is created whereby the mathematics situation alone generates the conditioned responses of fear and anxiety. It is in the conditioning stage that intervention must occur by placing mathematics within the context of a less threatening environment. Related to this model is the fact that students may generate negative self-talk concerning beliefs about their ability to do mathematics. The negative self-talk produces anxiety, leading to an inability to think, to avoidance toward mathematics, and ultimately toward an inability to perform mathematics. The inability to perform leads to additional negative self-talk and anxiety. Thus, there is a cycle that must be broken by helping students restructure their negative talk to positive means. This is the function of intervention programs.

Charlene Morrow (USA) addressed the segregation perspective, noting that we must create spaces where females can learn. Segregating males and females will not lead to improved learning opportunities if unequal resources or low resources are available for women. Often there are complications in getting appropriate resources to girls in mixed settings. Boys dominate discussions, and girls are often reluctant to speak up. Morrow cited an instance where a girls' mathematics club was established and this affected the quality of female comments heard in class. Several avenues of connection were discussed that affect females' participation in the mathematics environment. First, classes need to be structured as communities so that females feel a part of the learning environment and not as individuals on the outside. Second, because women often dislike an argumentative atmosphere, there needs to be an environment that is based on believing rather than doubting. For instance, a teacher might ask for further clarification of a response rather than indicate nonunderstanding of a response. Third, girls need support and encouragement to take on challenges to avoid the development of learned helplessness. Finally, students need to be given a voice toward discovery rather than always be told what is right or wrong.

The third model, the discipline perspective, was presented by SharanJeet Shan-Randhawa (UK). She emphasized the importance of awareness about barriers that hinder the learning of mathematics. Shan-Randhawa argued that mathematics lessons must be used to challenge attitudes and to raise issues that should be of concern to all students. For instance, mathematics lessons might deal with how mathematics relates to understanding the environment, jobs, and career opportunities. By focusing on problem situations that raise important issues, teachers can engage students in mathematics and enhance motivation.

The final theoretical model for addressing gender differences, the feminist perspective, was addressed by Sue Willis (AUS). She raised the issue that the mathematics curriculum must be rewarding, just, and emancipatory. There is a need for a curriculum that is nonsexist and that is inclusive of the experiences of women. The mathematics curriculum should be a tool for social critique and should be taught and used in a way that helps students understand the social world; students should be encouraged to consider where numbers describing a situation arise, how they are calculated, what assumptions underlie their use, and what uses are made of them. Further, the curriculum should help males and females identify those aspects that lead to different patterns of achievement and participation in mathematics.

During the panel discussion on cross-cultural issues, panelists discussed cultural factors that affect the mathematics education of females. Saleha Naghmi Havivullah (PAK) argued that the situation in developing countries is different from that in developed countries. Economic restraints that may not permit education for every student may be biased in favor of males. Remedial measures must consider the cultural situation of individual countries.

Neela Sukthankar (NG) gave a similar argument. In that country, there is a low percentage of women in higher education. Most educators are male, and so there are few female role models. Further, because much of the country is rural, it is often difficult to find good schooling. Male children are often sent to expensive private schools while female children are sent to local schools with poorer learning conditions.

Although not a developing country, Japan also faces problems in equity caused by the lower status of women in society and the traditional role models that affect women's expectations. Hanako Senuma (JPN) indicated that by the twelfth grade, boys outperform girls; while girls may be better in computation, boys are better on problems requiring higher-order thinking skills. Traditional roles in which men work and women stay at home serve to hinder female continuation in studying higher levels of mathematics.

In addition to societal factors influencing expectations, several panelists spoke about gender differences in mathematical performance. Paul Brandon (USA) indicated that girls in Hawaii perform better in mathematics than boys do. Furthermore, gender differences among nonwhites are greater than those among whites. Perhaps immigration to the West is more beneficial for females by elevating their status. Sharleen Forbes (NZ) reported that in New Zealand, gender differences in performance are found for the indigenous population while they have lessened for the population of European descent. Thus, there is the issue of what strategies used by the dominant culture are applicable to a minority culture. Finally, John Searl (SCO) indicated that in Scotland there are no significant differences in performance between males and females in mathematics performance, in terms of degrees awarded, even though such differences are present in England.

Overall, the study sessions of IOWME raised the levels of awareness about different factors that influence gender differences noticed in many countries of the world. Some of those differences can be addressed from a variety of theoretical perspectives. However, the cultural environment cannot be overlooked because economic and cultural influences do affect the expectations of women and their opportunities to pursue mathematical study.

UNITY AND DIVERSITY: SOME OBSERVATIONS ON ICME-7

Mark Saul
Bronxville School, Bronxville, New York

In a certain sense, we are all reinventing the wheel. The teaching of mathematics is so complex a process and so sensitive to the environment in which it occurs that we must constantly fine-tune our efforts, finding slightly different solutions to what might appear to be the same problem.

This phenomenon revealed itself in the two strands of ICME-7 that I followed most closely: Working Group 17 (WG-17: "Technology in the Service of the Mathematics Curriculum") and Topic Group 1 (TG-1: "Mathematical Competitions").

The talks given in WG-17 made it clear that the development of computer and calculator applications in the classroom has reached a plateau. Hannah Perl (ISR) has documented the introduction of graphing calculators in her secondary school classes. Most of her applications involved the solution of various types of equations by graphical methods—a type of problem that is by now a standard use of the graphing calculator. Perl gave anecdotes about individual students. She showed many examples of how the use of graphing calculators allow geometric and algebraic approaches to complement each other. Her observations made it *clear* that the use of graphing calculators in Israel differs slightly from how we use them in America.

Sharon Dugdale (USA) spoke about novel uses of her excellent package "Green Globs" and described observations of students working with the program. She presented new programs developed from her observations of students. Jenny Sendova (BUL) described a fascinating computer package, containing a complete language, which implements the construction of geometric models of physical and mechanical situations. A slightly different approach to the same problem ("Cabri-Geometry"), but without a programming language, was presented by Heinz Schumann (GER).

It was clear that the audience was comfortably familiar with the basic ideas discussed. This feeling contrasted with the experience of the ICME-6 meeting four years ago, where the very appearance of computers or calculators was a novelty.

Many teaching ideas that were then startling have now become classic software applications. The interest of the audience at ICME-7 seemed more focused on the details of the programs discussed and on exploring new aspects of existing software. If this is an accurate assessment, we have come quite far in only four years. We are now, to use Thomas Kuhn's words, "articulating a paradigm" of learning with computers and calculators rather than engaging in a technological revolution.

The presentations in TG-1 (mathematical competitions) also showed an evolution of thought and multiple solutions to common problems. ICME-7 saw a solidification of the membership of the World Federation of National Mathematics Competitions, an organization started at ICME-5. This year, the members voted to become a study group of the International Committee on Mathematics Instruction, an organizational step that reflects the continued development of international connections in this branch of our profession.

There was also continued development in the kinds of questions being asked about competitions. For example, Anthony Gardiner (UK) and Gilles Cohen (FRA) talked about how to develop contests that popularize mathematics and how to find activities for younger, lower achieving students. This aspect of mathematical competitions has not received as much notice as the national competitions that culminate in the International Mathematical Olympiad. Perhaps we are at the stage where these national contests are so well established that we can begin to use their prestige as a fulcrum to motivate other types of students in mathematics.

Several speakers talked about how competitions are related to more traditional educational activities. For example, Garnik Tonojan (ARM) spoke on the development of geometric thinking and its effect on other areas of thought through the solution of problems such as those presented in olympiad contests. Svetoslav Bilchev (BUL) gave a related talk, focusing more specifically on geometric transformations. Ali Rejali (IRAN) talked more specifically about how in his country competition questions and local curricula affect each other.

Harold and Betty Reiter (USA) gave a fascinating presentation of some of their work on gender and competition. It has long been recognized that mathematical competitions do not draw the best from young women mathematicians. The Reiters went further, to ask which sorts of questions favor girls over boys and how patterns of guessing might give clues to how girls work on competitions. The Reiters gave evidence supporting the hypotheses that girls have an edge over boys (in the junior high school years) in geometric problems and that they seem to take more risks in answering difficult problems. Whether or not these particular hypotheses are borne out by further investigation, it was generally agreed that future discussions should engage issues of gender in mathematics competitions.

A brief but interesting session was spent in planning for ICME-8. The audience was asked to suggest topics to be discussed four years from now. Many participants brought up questions relating to gender, race, and ethnicity as worthy of investigation. Another topic of interest, not addressed by speakers this year, was the role of profit in mathematical competitions. Who should run mathematics contests? Who should sponsor them, and toward what end? A third question asked involved the relationship between competitions and assessment. Should the outcome of mathematics contests be regarded as a judgment of the abilities of students? Can it be taken as evidence of the effectiveness of schools?

The final session of TG-1, called a "crossfire," was planned as a debate on the relative merits of competition in stimulating the learning of mathematics. The discussion, however, was not an occasion for controversy. Although some panel members objected to certain types of contest questions and pointed out limitations to the use of competitions, none of the panelists held positions that were strongly against competitions. Members of the audience did not see the use of competitions as controversial. The real value of this "crossfire" was that it brought out the uses of competition in different circumstances, how contests assume a wide variety of forms, and how they are able to attract an enormous range of types of students as participants.

This final session played upon a major theme running through ICME-7: variety and diversity in approaching a single problem. Perhaps we are all working toward the same end, and perhaps we could work more efficiently if we knew what the "correct" means toward that end might be. More likely, we are choosing different means, in keeping with the different cultures in which we live. This perspective shows mathematics education to be more like other human endeavors. Rather than steering inexorably toward our goals, we find successive approximations. Rather than using rationality to produce efficient and effective tools, we harness our social surroundings to accomplish our tasks. There is enough variation in our cultural environments to make the results noticeably different. Yet we have so much in common that each of us can learn from our neighbors. The ten days of ICME-7 gave us opportunities to do just that.

WHAT MATHEMATICS IS, WILL BE, SHOULD BE

Richard A. Gibbs
Fort Lewis College, Durango, Colorado

The main impression that ICME-7 has left with me is that the view of what mathematics is has definitely changed over the last decade (half-decade?). Ideas and recommendations for educators, which were previously accepted as theoretically valid but impractical to implement, are now being practiced or put into practice. There has been a true change in the thinking of mathematicians and mathematics educators alike about just what should be presented, to the general public as well as to students, as *mathematics*. I have the real feeling that mathematics is no longer seen as a subject ridden with "deadly drill," but is truly being viewed as an exciting and vibrant "experimental science" where students can explore, discover, and discuss patterns. Problem solving does indeed appear to be the central focus of school mathematics, and pencil-and-paper computation does indeed appear to be de-emphasized.

What has caused this change? The influence of the NCTM *Standards* cannot be underestimated, and the impact of technology cannot be ignored. The development of new methods of assessment (and, in some cases, the "rebellion" against traditional methods) has contributed to this change. The increasing popularity, worldwide, of mathematical competitions has contributed to the popularization of mathematics and a positive change, for the most part, in students' and the public's attitudes about mathematics.

My professional interests are in the areas of teacher preparation, discrete mathematics, problem solving and mathematical competitions. These interests were naturally reflected in my selection of activities during the Congress. A few memorable points from the sessions, points that led to the above impressions, follow:

In the opening session of Working Group 6, "Preservice and In-Service Teacher Education," NCTM president Mary Lindquist (USA) emphasized that the greatest need of our teachers is time. The teacher's time must be spent in ways to best enhance the student's mathematical experience. In the subgroup on preservice courses for K–4 teachers, Francoise Aberkane (FRA) stressed a problem-solving approach, pointing out that in many subjects, such as language or history, the answers to questions lie in the text or the memory, whereas in mathematics, answers come from reasoning and calculations. Stephen Krulik (USA) and Jesse Rudnick (USA), pioneers in the popularization of problem solving, described an in-service course that stressed problem solving and that involved teams of educators, administrators, and board members.

I found the sessions on mathematical competitions to be very stimulating. Speakers from many countries described the various competitions they sponsor. It was pointed out that a well-run competition can improve the public image of mathematics. The consensus is that competitions can provide opportunities for students to meet together, challenge themselves, pursue personal excellence, and appreciate the cultural value and beauty of mathematics. They definitely have a positive educational value when handled properly. The biggest concern expressed involves the perception of competitions by participants and the public. The concern is that competitions may be seen as vehicles that promote elitism and exclusivity and that they may be improperly used as assessment tools. There was also some criticism of competitions that use multiple-choice questions, although for competitions involving hundreds of thousands of students, no reasonable alternative to this format was suggested. Presently, very few competitions allow the use of calculators. Most of the organizations that sponsor competitions appear to have strong opinions, pro and con, regarding the use of calculators, and this issue is sure to receive much attention in the future.

Jon Coupland (UK) in his plenary session for the teacher education strand of the miniconference on "Calculators and Computers" emphasized that technology is continuing to advance, in spite of the objections of some educators and parents. It appears that the capabilities of future calculators will by limited only by how tiny the keys can be! Other speakers at the miniconference related how they have incorporated technology into their teacher-training courses and school classrooms. Thomas O'Brien (USA), who has developed a number of critical-thinking software products, asked "Should we spend years and years of a child's life to teach him or her to be very poor imitators of a $5 calculator?"

A good point made by Geoffrey Howson (UK) in the first plenary session was that it is unfortunate that we make the distinction between *mathematician*, *mathematics educator*, and *teacher*. At the appropriate level, all are mathematicians.

In the plenary lecture delivered by Maria Klawe (CAN), we were shown how, by using computers and video technology, elementary school children can be introduced to research.

The study session on the "Influence of Computers and Informatics on Mathematics and Its Teaching" produced my favorite quote from the Congress. Discussing the topic "Calculus, Continuous and Discrete," one panelist commented that "Calculus is a tool to use to find out why and when discrete models go wrong!"

Jack Van Lint (NLD) gave an interesting lecture stressing various techniques (graphs, 0–1 matrices, 0–1 sequences, minimal counterexamples) that can be used to unify various topics in discrete mathematics (which some feel ought to replace elementary geometry or calculus in the secondary school curriculum). Ron Graham (USA) also gave an interesting (and entertaining) presentation. He related that the current trends in combinatorics are (1) the construction of certain configurations (in contrast to just proving their existence) and (2) the development of efficient algorithms based on theoretical decision procedures. These trends are driven by the development of faster and faster computers.

Zalman Usiskin (USA) gave a provocative lecture titled "From Mathematics for Some to Mathematics for All." Tracing the historical development of movements that have led to changes in what mathematics is taught to all (societal need, development of efficient algorithms, available technology), he bluntly stated that paper-and-pencil computation is rapidly becoming obsolete and predicted that, indeed, those things that can be done quickly and easily by calculators and computers will eventually no longer be taught.

The last plenary lecture of the Congress was delivered by Benoit Mandelbrot (USA). Noting that people are fascinated by patterns, both mathematical and artistic, he advocated that mathematics be presented as an experimental science. Contrary to the view that drawings mislead and that "real" mathematics does not require diagrams, he posited that all mathematicians use diagrams when doing research and that we should "let the eye help the mind."

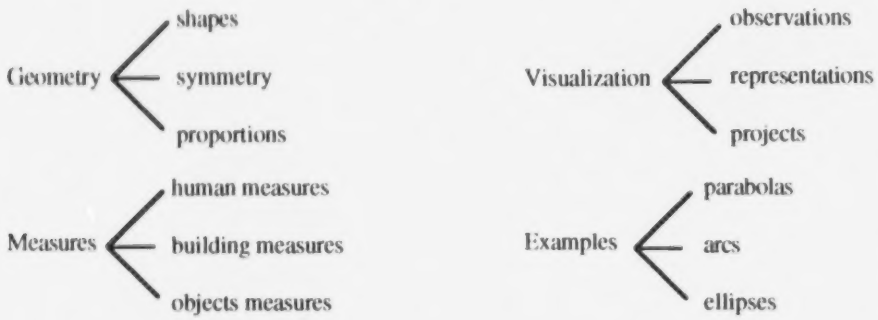
In conclusion, at ICME-7 I sensed the excitement with the current trends in mathematical education, discrete mathematics, technology, and competitions, and I had the opportunity to meet and exchange ideas with mathematics educators from around the world. I am optimistic about the future of mathematics education and the view of what mathematics is (and will be), and I'm anticipating ICME-8 where we'll be able to reflect on the effects of these current trends.

ART AND MATHEMATICS AT ICME-7

Catalina Reyero
Saint Martha School, La Puente, California

This paper is a personal reflection on the connection of art and mathematics at ICME-7 and an attempt to encourage anyone interested in the development of mathematics to attend ICME-8, which will be held in Seville, Spain in 1996.

Since full coverage of ICME-7 is impossible, I will sample the connection of art and mathematics from Topic Group 11. This group was organized by Rafael Perez-Gomez (SPN) to stimulate ideas on the relationships between mathematics and art. The presenters saw mathematics at school as being far from a creative activity and proposed to bring mathematical activities and artistic creativity together. Claudi Alsina's (SPN) topic was "House Work: Learning from Buildings." In presenting architecture as the most important art, he stressed structure, function, beauty, and people. He insisted that many models in mathematics may be drawn from ones around us. Buildings are an excellent source because they are real objects, are real models, have history, have real life inside, and present problems. Buildings can be used in teaching as follows:



Ronald Brown (UK) presented "Mathematical Themes in John Robinson's Universe." He reported the use of very simple forms in sculpture such as the sphere and the circle and shapes cut from an oval shape. What struck me the most was seeing children looking happily at the abstract forms and feeling comfortable. Could this type of art be a proper visual representation for teachers of mathematics?

Ceferino Ruiz Garrido (SPN) developed the theme of "Sculpture and Geometry," and described geometry as form, object, model, and structure.

In "Learning Maths in the Alhambra of Granada," Rafael Perez-Gomez (SPN) divided his presentation into four parts: the Nasrids' use of geometry; color; M. C. Escher; and crystallographic plane groups. The Nasrids transformed geometry into a graphic language, used geometrical shapes in abstraction, made use of a single pattern in geometrical decoration, and used the representation of the unit of any decorative composition that, when multiplied, could cover the entire surface area. Nasrids also applied color to their polygons to cover the sides so that only the decorations applied over them were seen and to obtain different mosaics from the same design.

Activities based on the creation of mosaics give us the opportunity to introduce a great many subjects into mathematics classrooms, such as the study of polygons, the study of semiregular mosaics, the introduction of kaleidoscopes, and the study of mosaics in three dimensions. Mathematics educators can provide the connection between art and mathematics by using fabric designs and patterned paper, kitchen and bathroom floors, and the art world in general. Escher was inspired to create his famous "Regular Division Plan" after visiting the Alhambra. Even though Escher added animals and people to his mosaics, the main component still remains the same—quadrilaterals, triangles . . . —in other words, MATHEMATICS!

The study group was an inspiration to me because I am very interested in the integration of mathematics and art as a means of developing logic and understanding in problem solving, arousing interest in potential drop-out students, and developing creativity in today's generation.

IMPRESSIONS REGARDING THE PROGRAM OF ICME-7

Karen Sharp
C. S. Mott Community College, Flint, Michigan

The opening day of the ICME-7 Congress was highlighted by a miniconference on "Calculators and Computers." There were five strands from which to choose: 5- to 11-year-olds, 11- to 16-year-olds, 15- to 18-year-olds, mathematics undergraduates, and teacher education. Each strand began with a plenary session to set the stage, followed by a choice of two workshops from a multitude of choices. For example, in the "Mathematics Undergraduates" category, choices could be made from a vast array of software and computers (Derive, MicroCalc, TI-81, Geometer's Sketchpad, Maple, Mathematics) as well as special programs and pedagogical approaches using the technology (laboratory methods, interactive textbooks, constructing abstract mathematical concepts, visualizing theorems of analysis, exploring, project CALC, and assessment). The wide variety of information presented is obvious. Furthermore, world leaders in each of these areas were the presenters.

I participated in the working group on distance education. Topics covered in the four days and a brief summary of the material considered follow:

- *Course Development of Written Materials.* Major consideration should be given in designing distance education material to: the cultural context of the students, the presentation of mathematics both formally and informally, the examples and nonexamples presented to the students by the materials, as well as the ones to be worked by the students, and the assessment tasks used to stimulate a thorough learning approach by the student.
- *Role of Technology in Distance Education.* Historical barriers to effective mathematics learning are constantly being erased by improvements in technology, and examples of learning that is *more* effective through distance education than through traditional classroom instruction are emerging.
- *Communications with Students.* Communications with students in distance education settings occur in a wide variety of traditional and innovative technologies.
- *Assessment and Research.* Distance education topics that should be researched, research methods, and international research cooperation were discussed.

Plans were made to begin an international newsletter on distance education.

I also attended the sessions of IOWME, the International Organization of Women and Mathematics Education. There were two general sessions. One was a panel discussion on theoretical models for addressing gender differences in mathematics and the other was a panel discussion on cross-cultural issues. The other two meetings of IOWME allowed a choice from among eight small-group discussions of the two panel topics. The sense of camaraderie and acceptance, along with the enthusiasm of sharing common concerns of women and different solutions to problems, was something special.

From the seventeen topic groups, which met for two 90-minute sessions, I chose to attend the "TV in the Mathematics Classroom" topic group. There the participants discussed the essential ingredients in broadcasts and videos suitable for classrooms, ways in which teachers can enhance such material, optimum procedures for obtaining feedback, and optimum procedures for making known what is available.

In addition to the more structured portions of the program, less structured opportunities were provided for meeting and interacting with mathematics educators from across the world. One evening I met with leaders of national mathematical organizations. Unfortunately, there wasn't time for the approximately twenty different groups to do more than introduce themselves and talk briefly about the activities, members, and member services of their organizations. Plans were made to provide for closer communication among the groups between now and ICME-8 in Spain.

MISCONCEPTIONS

Marilyn Mays
North Lake College, Irving, Texas

One of the best features of ICME-7 was the opportunity to meet with the same group of people each day for several days to discuss a chosen topic. This format offered the advantage of exploring topics in depth with the added benefit of becoming acquainted with a small group of individuals from around the world.

My choice was the working group considering the topic of "Students' Misconceptions and Inconsistencies of Thought" led by Shlomo Vinner (ISR). Conventions were agreed on at the onset.

1. Inconsistent thinking is observed in a person who believes in p and q from which r and the negation of r can be derived.
2. Compartmentalization may have more than one form. One example is the failure of a certain stimulus to evoke a certain associated response or for a stimulus to evoke an incorrectly associated response.

A student who incorrectly declares that

$$\sqrt{a+b} = \sqrt{a} + \sqrt{b} \text{ because } \sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

is compartmentalizing. In a similar manner, the student is compartmentalizing who sees no relationship between the equations

$$x^4 + 8x^2 + 16 = 0 \text{ and } x^2 + 4x + 4 = 0$$

Vinner contended that students' misconceptions lead to inconsistencies when the instructor's cognitive structure is imposed on the student. An example of the ineffectiveness of the latter can be found in the student's failure to appreciate the implications of a counterexample to a cherished belief. To teachers it seems obvious that it is not true that

$$\sqrt{a^2 + b^2} = a + b$$

simply because that leads to the contradiction

$$\sqrt{4^2 + 3^2} = 7.$$

However, this often does not deter a student from using the incorrect formula. To the student, the stimulus, the contradiction, does not seem related to the formula, and therefore, it is not evoking the expected response, rejection of the incorrect formula. This lack of association of two related items creates misconceptions. The contradiction to reality may not create any enlightenment because students do not expect mathematics to make sense.

This example, in a small way, demonstrates the teachers' misconceptions about what is an effective teaching tool. The teacher is using a basic tenet, "Math is logical," to teach a concept to someone who not only does not have that basic tenet but for whom the basic tenet would be difficult to accept.

An interesting dialog developed between two members of the group about an effective method of teaching the concept of square roots mentioned above. One teacher said that she used the right triangle to explain this concept to her students, and they seemed to grasp the concept with little difficulty. Her explanation was as follows:

For the right triangle with sides a, b, and c,

$$a^2 + b^2 = c^2 \text{ and } \sqrt{a^2 + b^2} = c. \text{ If } \sqrt{a^2 + b^2} = a + b, \text{ then } a + b = c.$$

The latter is a contradiction because the figure with sides a, b, and c would not be a triangle but a straight line.

Another member of the group expressed the pessimism I was feeling about students' understanding the multi-step explanation, much less being able to apply it. A comparison of the teaching situations of the two gave at least a partial explanation of their disparate perceptions about the success of this method. The proponent of the right triangle method worked with highly motivated returning adults in one-on-one

tutoring sessions and had little opportunity to observe their ability in the classroom to generalize the concept to other situations. The more pessimistic member of the group taught recent high school graduates in a college classroom situation. It is clear that overgeneralization about the effectiveness of a teaching method could lead to inconsistencies in pedagogy.

Glenda Lappan (USA) addressed teachers' misconceptions in her lecture, "Training Teachers or Educating Professionals? What Are the Issues and How Are They Being Resolved?" One popular misconception that is being perpetuated in colleges of education in many parts of the world is that teachers learn how to be good teachers by being told. Lappan also stressed the importance of helping teachers challenge their misconceptions about students' abilities to learn. Her research indicates that teachers often are quick to write off some very capable students because of stereotypes or behavior patterns.

Students' misconceptions about their abilities in mathematics was a recurring theme in one meeting of the International Organization of Women in Mathematics Education. Several of the speakers emphasized that one of the principal obstacles to overcome in mathematics-anxious people is their conviction that they are incapable of learning mathematics. Panelists from four different countries discussed their efforts and techniques for altering students' self-perceptions.

The opportunity to participate in ICME with people from all over the world was afforded me by a grant from the National Science Foundation through NCTM. Through these avenues we may learn to share mathematics with more people. One valuable lesson that will guide me in the future is, "Teachers' misconceptions about students and the way they think may be as great a hindrance to our students' learning as are the misconceptions of our students themselves."

A SAMPLING OF INTERNATIONAL PERSPECTIVES AT ICME-7: WE'RE ALL THE SAME, BUT DIFFERENT

Eric W. Hart
Maharishi International University, Fairfield, Iowa

International camaraderie was in great supply during the one-week Congress, as were interesting conversations and thought-provoking presentations. Presented here is a sampling of some of the international perspectives I encountered.

Heuristics and Problem Solving

The poster session area at ICME-7 was the location of many animated conversations. The main issue concerned general heuristics versus domain-specific strategies and knowledge. There seemed to be agreement that the teaching of general heuristics per se has not been shown to be an effective way to improve problem-solving performance. Rather, the conceptual schema possessed by a student is the key factor in determining success in problem solving. Interdependencies between content understanding and process use begin to emerge as salient, and in fact, we could even say that processes, metacognition, and misconceptions are actually part of one's conceptual schema.

Proof

What is proof? How should it be taught? Is the notion of proof different for different cultures? These were some of the questions dealt with in the topic group on proof and in other presentations.

Josette Ada (FRA) proposed that the transition from empirical to theoretical reasoning is not natural, and thus it is the teacher's responsibility to facilitate this transition. She felt that proof in the typical classroom setting jumps too quickly into theoretical reasoning in a way that is formal, rote, and without understanding. In a similar vein, Erich C. Wittman (GER) pointed out the distinction between meaningful representations, which relate to informal proofs, and verbal representations, which relate to formal proofs. He claimed that meaningful representations can and should be taught early. In a general discussion of these and related ideas, there seemed to be consensus that one certainly needs precision in proof but also clarity. And we should teach proof early, make it exciting, and use appropriate proofs.

But what is proof? Judith Grabiner (USA) gave several examples of changes in the practice of proof over time. Philip Davis (USA) discussed what he considers to be major principles or components of mathematical evidence. These principles include induction, experimentation, intuition, applicability, computation, appropriateness, aesthetics, convenience, and authority.

George G. Joseph (UK) contrasted styles of argument in two different cultures—India and the West. He began by comparing traditional Indian mathematics, the earliest records of which are found in the Sulba sutras of the vedic literature, to ancient Greek mathematics. He claimed that Indian mathematics is algebraic, algorithmic, empirical, utilitarian, and has no formal proof. In particular, in Indian mathematics there is no reference to fixed axioms, no formal deduction, and methods of indirect proof can only be used to show the nonexistence of mathematical objects. This is in stark contrast to Greek mathematics, which is geometrical, anti-empirical, and attaches great value to formal proof. Furthermore, the ancient Greeks were intolerant of applications, claiming that mathematics was derived from philosophy and was the realm of "disembodied and abstract thought," whereas the Indian view is that mathematics is derived from the study of linguistics and is of the essence in all areas of life, including, love, music, dance, geography, astronomy, meditation, religious practice, and cooking. Joseph concluded by proposing that the two cultures, India and the West, have differing views of the nature of mathematics—in the traditional Indian view, mathematics is a set of useful rules, whereas in the West, mathematics is seen as a body of infallible truths.

Humanistic Mathematics

Thomas Tymoczko (USA) also talked about the nature of mathematics in his presentation on the philosophy of mathematics. He discussed the humanistic, pure, and utilitarian aspects of mathematics. Traditional (western) philosophies of mathematics have focused on the existence of mathematical objects and have viewed mathematics as pure and a priori. The newer utilitarian viewpoint sees mathematics as being inextricably linked to human activity—business, trade, war, and so forth. In fact, it is claimed that mathematics and human practice are so inextricably linked that one cannot exist without the other. From this point of view, pure mathematics is seen to be extracted from human practice. As for the existence of mathematical objects, they are seen to be just as real as common sense objects. For example, since dogs and money exist and mathematics quantifies over dogs and money, mathematical numbers must also exist. Tymoczko's objection to utilitarian mathematics is that it leaves out the intrinsic beauty and unity of mathematics, it ignores mathematics for its own sake, and it does not take into account the historical context of mathematics.

Tymoczko claims that mathematics is more than the abstract study of mathematical objects (pure mathematics) and that it also goes beyond being just utilitarian. The mathematical universe is too rich and complex to be captured by either of these viewpoints. Instead, mathematics as a discipline consists of the mathematical universe along with a human perspective. The human perspective includes historical context, aesthetics, and an interest factor—mathematicians study mathematics that is interesting to them. Tymoczko calls this viewpoint *humanistic mathematics* and proposes it as the true nature of mathematics.

Conclusion

ICME-7 was a clear demonstration of the vitality and coherence of our profession. There was uncanny consensus about the problems we face and the directions we need to go in to begin solving those problems. The differing international perspectives only seem to complement one another as we work together to improve mathematics education around the world.

PERSONAL OUTCOMES FROM ICME-7

Bernadette H. Perham
Ball State University, Muncie, Indiana

ICME-7 themes and discussions reinforced an ever-increasing presence of technology in the school mathematics curriculum and the importance of mathematics in the global marketplace. From Geoffrey Howson's (UK) opening plenary lecture on "Teachers of Mathematics" through the miniconference on "Calculators and Computers" to the working and topic group meetings, the Congress reaffirmed my belief that the well-being of mathematics education rests with teachers.

Congress participants were invited to join a working group and a topic group. These groups met regularly during the remaining days of the Congress. In support of my interests, I joined the working group "Technology in the Service of the Mathematics Curriculum" and the topic group "Constructivist Interpretations of Teaching and Learning Mathematics." In addition to these, I attended a three-session presentation of the International Commission on Mathematical Instruction Study, "Popularization of Mathematics." These sessions made me aware of the variety of ways that mathematics has been popularized in other countries, namely, Australia, Great Britain, and France, as well as in the United States. Perhaps the strategies used in other countries could be used to a greater degree in our own.

The sessions on "Popularization of Mathematics" (Henry Pollak, organizer) covered topics on the use of public events, television, and films. (I shall describe some, but not all, of these topics.) At the beginning of these three sessions, Dudley Blane (AUS) addressed the topic "Mathematics Trails.? One purpose of the trails is to stimulate children's interest in mathematics. Mathematical activity outside the formal classroom infiltrates what goes on in the classroom. While having fun on the trails, children engage in discussions that promote mathematical language development and the exploration of mathematical ideas in real world settings.

Most mathematics trails are for the general community. Blane described in detail a Family Maths Trail in Melbourne that was sponsored by a local newspaper. Eight thousand copies of the trail were distributed to participants who ranged in age from six months to eighty years. Walking leisurely along the trail, families estimated and calculated the height of lamp posts, determined the value of pi, figured the cost to send a letter, explored the Fibonacci sequence in nature, and determined the number of windows in a building. The trail was noncompetitive. There were no prizes at the end of the trail, but there were a lot of smiles.

The second session began with the topic "Televising Mathematics: Popularization or Distortion," presented by Celia Hoyles (UK). Hoyles, a mathematics professor, and Rob Buchman were the show hosts in this prime-time Yorkshire Television show "Fun and Games" (1987–90). Buchman presented puzzles, and Hoyles gave hints and suggestions for a mathematical framework. Hoyles asserted that the popularization of mathematics was all about extending mathematical culture. Many believe that mathematics is school bounded and has little to do with the outside world. Furthermore, it is a popular belief that studying more math involves doing longer sums. The TV show made mathematics more visible in our culture.

"Fun and Games" was billed as a mathematics show, and its viewing audience numbered up to 10 million. The show was noncompetitive—there were no prizes. The puzzles were nontrivial and their complex solution strategies were counterintuitive. Ordinary people who were confident in solving puzzles but not necessarily confident in mathematics nor knowledgeable about the formal language of mathematics were the contestants in this unrehearsed show.

In a survey, 20 percent of those who viewed this series said that as a result, they felt differently about mathematics and had a different view of mathematics. On hearing these results, one of the ICME participants countered with the questions: "What is mathematics? Does popularization strengthen or distort mathematics?" Left to ponder these questions, the participants concluded the session by watching mathematics films in the presentation by Jean-Michel Kantor (FRA), "Les maths sympas grâce à la télé?" Later that evening, additional mathematics films by French artists were introduced by Kantor in his

presentation "Les maths c'est du fun!" The films leave one with the message that mathematics is alive and relevant to modern culture.

The third and final session included a presentation by Florence Fasanelli (USA), "Math on the Washington Mall." Fasanelli, Frederick Rickey, and Richard Thorington developed a self-guided all-day mathematical tour along the Washington Mall. The tour, like the mathematics trails described by Blane, is designed to sensitize one to see that mathematics permeates daily living. The tour begins at the Capitol, goes past the Washington Memorial, and ends at the Lincoln Memorial. One of the first stops is the Whispering Gallery in Statuary Hall, where the tourist explores this mathematical phenomenon. Proof is not an integral part of the tour, but the guidebook points out that an original proof of the Pythagorean theorem is attributed to President Garfield. The tourists do classifications of tilings as they walk along the route, and they consider the causes of the rise and fall of the tide. While in the National Academy of Sciences, they view a bronze statue of Einstein as well as representations of other mathematicians. In the Jefferson Memorial they ponder the shape of its dome.

The general population has a difficult time knowing what mathematicians do. Perhaps the Popularization of Mathematics movement can bridge the gap between mathematics and how it is understood by the general public.

EMERGENT THEMES AT ICME-7

Diana V. Lambdin
Indiana University, Bloomington, Indiana

ICME-7 in Quebec was only my second opportunity to attend an international mathematics education conference. At various times in my week in Quebec I was enthralled, overwhelmed, excited, bored, exhausted, stimulated, and challenged. I attended a dizzying variety of sessions and activities but also took to heart the advice of more experienced friends and colleagues to reserve time for informal interactions with old and new friends from around the world.

ICME-7 provided such a wealth of information and opportunity that it is difficult to synthesize it all, but I have attempted to identify a number of themes that I noted emerging throughout the conference. Each conference attendee could make his or her own list of emergent themes, and these lists might differ considerably from mine. My themes were derived from the subset of activities that I attended, which included (1) Working Group 6 on "In-service and Preservice Teacher Education," (2) Topic Group 14 on "Cooperation between Theory and Practice in Mathematics Education," (3) lectures by Geoffrey Howson, Glenda Lappan, Maria Klawe, Jan de Lange, and Seymour Papert, (4) a film featuring George Polya—*Let's Teach Guessing*, (5) a specially organized discussion on evaluating preservice elementary teachers, and (6) numerous informal discussions. What were the themes I heard emerging? I'll deal with each briefly.

Time as a Limiting Factor

Numerous wonderful suggestions for improving the teaching of mathematics were shared at ICME sessions, but inevitably, it seemed, someone would raise the issue of time. Is what we are dreaming that teachers may accomplish possible given their pressured schedules? For example, portfolio assessment was repeatedly touted as a new and promising development, yet it takes only a moment's calculation to figure that if a middle school teacher with six classes a day of thirty students each spends just five additional minutes per student each week as a result of portfolios, she will have increased her weekly workload by fifteen hours!

"Covering" versus "Uncovering" Mathematics for Students

Teachers also feel pressures from students, parents, peers, and administration to "cover" a body of material. Is a fifth grade teacher derelict in his duty if he never "gets to" the chapter on probability? (Or what if he never "covers" decimals?) The NCTM Standards make it quite clear that students need to be personally involved in active investigations if they are to develop conceptual understanding. In fact, it has become almost a buzzword to say that students must "construct" their own meanings for mathematical ideas. The job of the teacher, then, becomes less one of covering particular chapters in a text and more one of helping students uncover mathematics for themselves by providing appropriate activities, feedback, and opportunities for conversation and reflection. The issues of *time* and *covering* are obviously interrelated. Because it takes less time to cover than to uncover material, and time is of the essence in schools, we are often tempted to cover a topic that we do not have time to deal with in depth. Are we doing students a disservice?

Varieties of Notions of What Mathematics Is

ICME-7 provided an opportunity for numerous debates about the nature of mathematics. What made the debates interesting was that the participants brought such diverse points of view. Included at the meeting were philosophers, teachers, curriculum developers, teacher educators, researchers, and those of us who from time to time have tried our hands at each of these tasks but consider ourselves novices at most of them. What is mathematics? I won't attempt to summarize the debates here. Let it suffice to say that one's

viewpoint about the nature of mathematics inevitably colors one's position on a wide variety of other issues including what mathematics should be taught and how it should be taught, what constitutes research in mathematics education, how teachers should be educated, and so on.

What Is Mathematics Education Research?

Another hotly debated topic at ICME sessions was the nature of mathematics education research. The dominant research methodologies in our field have been changing in the last decade or two and there seems to be no simple answer to the question of what constitutes good research. One notion that I heard repeatedly was that research should be viewed as a process rather than as a product. We may do research at least as much for the learning that takes place in doing it as for the results that it may produce. Another debate concerned the appropriateness and efficacy of narrative, or story telling, as a form of research. Moreover, in both the teacher education working group and the theme group on theory into practice, I repeatedly heard a call for classroom teachers to become more involved in mathematics education research. If changes in schools are to take place, classroom teachers must be involved in designing studies that will help all of us understand more about how students learn, how teachers teach, how classrooms function, and so on.

Changing the School Scene—How to Accomplish This

In the working group on preservice and in-service teacher education, I not only learned quite a bit about how teachers are educated in various countries around the world but also listened to accompanying debate on how teacher education as well as schools of the future must change. There seems to be agreement that changes are necessary but also a healthy respect for the political and social realities that we inevitably encounter in trying to implement change. Sharing ideas about innovative programs around the world and the conditions under which they have been implemented is one way to find out how change is possible. ICME-7 provided a forum for such information sharing.

The themes above provide an overview of some of the issues I've been thinking more about since my attendance at ICME-7. At the meeting, I encountered these ideas not only in formal sessions but also in a wide variety of informal interchanges. In the final analysis, it was probably from these person-to-person contacts that I learned the most—from late-night reflection in a spartan dormitory room and from chats with new-found friends over breakfast in the university cafeteria; from a day-long discussion with friends on a tour bus riding through the French Canadian countryside and from brief interchanges with strangers sitting next to me in lectures; from informal talks over giant hamburgers at the outdoor barbecue and from sharing research ideas in a sidewalk cafe in old Quebec on a moonlit August evening. The value of an international meeting is in the people one meets and in the ideas one shares with them.

REFLECTIONS ON ICME-7

Jerry P. Becker
Southern Illinois University, Carbondale, Illinois

In many respects, ICME-7 represented the changing preoccupations of mathematics educators around the world. For example, there was special emphasis on technology (computers and calculators); the theme "Mathematics for All" was prevalent; the political, educational, social, cultural, and economic realities in which mathematics education exists were obviously evident; and there was an excellent social program that provided opportunity for professional dialogue on a personal level. The Congress also provided opportunities for conferees to engage in sustained dialogue on a personal basis or through participation in discussions built into the working, topic, and study groups and other components of the scientific program. With each ICME, I have observed important strides being made towards establishing mathematics education as a discipline in its own right.

I participated in all the Working Group 6 sessions as a subsession chair, speaker, and participant. There were a number of very interesting papers and discussions in these sessions. In fact, there were a number of convergent issues and problems stemming from the presentations and discussions.

To begin with, however, it seems widely accepted that teachers need to emphasize problem solving and mathematical thinking as the most important goals in teaching mathematics. Conferees also agree that classroom environments are needed in which students are actively involved in constructing their mathematical knowledge.

Although it was not overtly addressed, the tension that exists between those who advocate mathematical content knowledge and those who advocate pedagogical knowledge was evident in some paper presentations and discussions. All agree, however, that those teachers who are most effective both possess strong mathematical backgrounds and know a lot of pedagogy, too. However, the proper balance or integration of the two into the reality of the teaching populations in various countries is certainly still open to further examination. Further study is required, and perhaps there should be a focus on this issue in the next Congress. Development of a strong position statement by the international mathematics education community seems in order, in my view, in which the importance and role of the mathematics community in preservice and in-service education of teachers is emphasized.

At least three types of intervention projects were discussed in the working group sessions I chaired: (1) Using a holistic approach in which all mathematics teachers and students in a school were involved, with all aspects of the curriculum being addressed (e.g., textbooks, lesson plans, testing, teaching approaches, staffing divisions, administrative procedures in the school, and evaluation of teachers and student learning). A proteaching mode was used in this approach that involved teacher observation and feedback, demonstration teaching, and regular in-service education sessions. (2) Summer residential institutes in which mathematics teachers were exposed to innovative work being done by the teaching faculty, with the expectation that teachers will carry their new knowledge not only into their own classrooms but share it with other teachers. Here, teachers were regarded in a big way as *agents* of change. (3) Design and implementation of a problem-solving course for teachers, taught by university faculty.

A number of themes emerged during the discussions that were regarded as centrally important in in-service education:

1. *Follow-up of teacher participants* is critically important. The follow-up may be immediate, as in the case of a consultant working with teachers in their school on a regular basis, or it should occur as soon as teachers begin implementation work in their classrooms following a summer institute. Whichever is appropriate, the follow-up is crucial and the sooner the better.
2. *Demonstration teaching* seems to play a crucially important role in providing the glue-to-the-classroom for innovative work. It is one thing to simply lecture to teachers but quite another to (a) model the desired teaching in the in-service programs, (b) model the desired teaching with school students, with teachers observing, and (c) have teachers teach demonstration lessons to school students with other

teachers observing. Evidence was provided that the latter holds great potential for classroom realization of innovative work.

3. Teachers need *curriculum materials in hand* to use in classroom implementation following their in-service institutes or workshops. Further, it is important that teachers have a hand in developing or adapting the materials for use in implementation—a sense of ownership may play an important role in successful implementation.
4. Bringing teachers together to study teaching and curriculum improvement leads quite naturally to a *sense of collegiality* that extends well beyond the in-service institute or workshop. This collegiality may play an important role in successful integration of new content and pedagogy in teachers' classrooms.
5. There is a need for a *strong commitment of school administrators* to support their teachers in implementing the objectives and work of in-service institutes. Securing the commitment may take various forms, for example, discussions with administrators about the project, the role of their teachers in the project, and its implications for classrooms in their school(s), followed by firm support in practice; or administrators signing teachers' applications to participate in a project, thus formally signifying a commitment of support.

Finally, the question of assessment of student learning arose quite naturally in discussions. One approach presented was based on the work of Japanese researchers in classroom implementation of their open approach to teaching mathematics, namely, (1) using problems that have a unique answer but many ways to find the answer (here the *process* is open); (2) using problems that have many correct answers, for example, problems that involve finding many rules or relations, classifying, and measuring (here the *end product* is open); (3) having students pose or formulate problems of their own after they have solved a given problem, using analogy or generalization (here the *way to develop problems* is open) (Shimada, 1977; Becker, 1993).

Assessment then could involve (1) a measure of *fluency* in students' mathematical thinking, in which the number of different ways to solve a problem or number of different correct answers is quantified; (2) a measure of *flexibility* in students' mathematical thinking, in which the number and quality of different mathematical ideas of students in solving a problem are determined; (3) a measure of *originality* or *creativity* in students' mathematical thinking, in which unique ideas of a student (not found by other students) are noted; and (4) a measure of *elegance*, in which the degree of elegance in students' mathematical *expression* of their thinking is assessed. Assessment of these four components could be based on an examinination of students' worksheets and on observation of students (a) while they are working individually or in small groups and (b) while the teacher leads a comparison and discussion of ways (for their mathematical quality) in which students solve a problem (Shimada, 1977; Becker, 1993; Nagasaki & Becker, 1993).

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MY FIFTH DANCE AT THE ICME BALLS

Thomas J. Cooney
University of Georgia, Athens, Georgia

Whenever one attends an international conference on mathematics education, it becomes clear that culture and context are an integral part of how the field is interpreted. That is one of the lessons I learned from attending international congresses of mathematical education. Although the variance among individuals within a particular country may be greater than the variance across countries, the influence of culture is undeniable. In this sense ICMEs have a social dimension as well as a scientific one. Thus is seems particularly relevant not only to talk about the scientific part of ICMEs but also to consider the context in which this scientific part of ICME's programs occur. This is not to say that ICME is essentially a social event, far from it, but it is to acknowledge that society and context shape our perceptions about mathematics education.

ICME-7 was my fifth ICME; I missed the first two. The Quebec experience added yet another lesson on the importance of culture as I experienced firsthand, both within the Congress and in visiting old Quebec City, the people's emphasis on a bilingual society. It was more than just reading menus in sidewalk cafes in delightful old Quebec City on a warm summer night. It had to do with the nature of the interactions in the working groups, the kinds of questions raised, and the importance of communicating across cultures. In the working group for which I had organizational responsibilities (The Public Image of Mathematics and Mathematicians), this communication was made easy by the translations of Jean Dionne, a professor at Laval University in Quebec and local organizer for our working group. Although the fifteen presenters in our working group represented ten different countries, the bilingual communication in French and English was an essential part of the proceedings—unlike any of the previous congresses that I had attended.

The presentations in our working group highlighted concerns or approaches that seemed to be culture-specific, yet many characterize the American scene as well. For example, Elizabeth Oldham (IRE) discussed the limited view of mathematics that many of her preservice elementary teachers exhibited. Fou-Lai Lin (TAI) and his colleagues talked about their concern about "exam math," namely, that so much of mathematics education in Taiwan is determined by students taking tests on an almost daily basis. Toshiakira Fujii (JPN) presented a study on parents' perceptions of mathematics and how they differ from those of their children. For example, 91 percent of the Japanese parents think that mathematics is a difficult subject to succeed in if once it is failed; yet only 66 percent of the eighth graders hold a similar view. Japanese parents hold teachers in high regard and have high expectations of teachers, as indicated by this result: 92 pecent of the Japanese parents think that "mathematics could be understood well if it was taught properly." Students were not so sure; only 79 percent of the eighth graders agree. Furthermore, 91 percent of the parents think that "in mathematics you can get the answer clearly and finally," a point agreed to by only 70 percent of the Japanese eighth graders surveyed. Other presenters discussed their efforts to influence students' images of mathematics ranging from programs like Family Math in the United States to Math Trails in Australia in which students use the environment to create and explore mathematical questions. These are just a small sample of how people from various cultures approach mathematics education.

Although the cultural differences revealed in international congresses are evident and represent an important aspect of understanding mathematics education, there are also many similarities. Despite different contexts, issues surrounding the teaching and learning of mathematics and teacher education are often strikingly similar. For example, concerns expressed about teacher education seemed to have more in common than they had differences—as did students' understanding of different topics. Assessment also attracted considerable attention in many countries, although perhaps not as much as in the United States.

If I were to address the question of how ICME-7 differed from its four predecessors that I attended, I would focus on the influence of the French—who I think have made significant progress in mathematics education, the use of technology, and the attention given to the teaching of mathematics. The increased emphasis on technology over the last four ICMEs is hardly a surprise. Although the work in the United

States is still leading the way, other countries are doing significant things. Colette LaBorde's (FRA) plenary address on technology demonstrated recent developments in France.

Perhaps the most surprising emphasis was the attention given to the teaching of mathematics. This perception may only be a factor of the sessions I attended—I usually select sessions that focus on the teaching of mathematics or teacher education. Still, one of the plenary speakers, Geoffrey Howson (UK), focused explicitly on issues related to the teaching of mathematics. Other speakers also addressed issues related to teaching with respect to technology, assessment, and cultural diversity. Although one might expect this at a conference on mathematics education, the emphasis on teaching seemed more evident than had been the case at previous ICMEs. Researchers now seem to be taking seriously the problems teachers face in trying to realize the many facets of reform.

Four years ago I edited a publication entitled *American Perspectives on the Sixth International Congress on Mathematical Education*, a publication similar to this one. Some authors were critical of the substance of ICME-6. I had the impression that this time around the presentations and discussions were more substantive and were more likely to be based on empirical evidence than was the case at ICME-6. I thought that ICME-5 (1984, Adelaide) represented a turning point in terms of mathematics education becoming more scientific (broadly interpreted) in its orientation. I am not sure that I sensed that same growth at ICME-6 (1988, Budapest), but it was clearly evident at ICME-7. I believe we are making progress in understanding issues related to the teaching and learning of mathematics and related issues involving teacher education. It will be interesting to see whether research (and development) on the central issues of the field continues to mature. I look forward to the possibility of making that assessment and attending my sixth ICME in Seville in 1996.

MATHEMATICIAN, MATHEMATICS EDUCATOR, TEACHER: THE INTERSECTION IS NOT EMPTY

Genevieve M. Knight
Coppin State University, Baltimore, Maryland

I arrived at Université Laval in Quebec on 17 August to attend my second International Congress on Mathematics Education. My colleagues from all over the world who are interested in the teaching and learning of mathematics had the opportunity to listen; react; share; make formal presentations; and exchange ideas, beliefs, and information. My academic preparation, interest, and current professional responsibilities focus on mathematical content and the preparation of teachers of mathematics. I selected the "Preservice and In-Service" working group because this is my current interest and the "Constructivism" topic group because I find the literature a bit "fuzzy" and wanted to be able to listen to the experts and ask questions. A typical ninety-minute working group session might have as many as twelve separate subgroups addressing issues such as preservice courses in mathematics, multicultural education, alternative assessment, mathematics specialists K–5, and staff development, and a typical topic group might have two speakers and a discussant. The intended fabric of the Congress was "small group interactions." The format of the preservice and in-service working group enabled all persons to share their views, gain additional information, and ask specific questions. Often my peers had specific issues and concerns to raise. However, usually we shared similar concerns about resources, size of the teacher candidate pool, certification, national trends, and educational reforms.

The first plenary lecture was given by Geoffrey Howson (UK) who set the tone for the Congress. Howson challenged attendees to see mathematicians, mathematics educators, and teachers as one unit, not the union of mutually disjoint sets but an intersection of mutually overlapping sets. He cited an 1839 publication that described the difficulty of recruiting teachers. Now and then the reasons why are the same! Are you able to see the *now* in the *then* set? *Then* = {low place in the public's eye, poorly paid, not recognized as equal to other professional groups, low self-esteem}. We were challenged to (a) never be convinced that teachers do not have a role in determining what mathematics content and pedagogy ought to be, (b) ensure that the mathematicians see themselves as partners in the educative process and work cooperatively with the educators and classroom teachers, and (c) remember and learn from the advertisement "Wanted: a swimming teacher who can swim." Rephrase the saying to indicate that teachers of mathematics must know mathematics. The words of Howson's lecture lingered in my mind throughout the entire Congress. As I reflect on the various activities that I participated in, I find myself visualizing the intersection of Howson's three sets. If the intersection is indeed empty, then the players with major roles in mathematics education are in grave trouble. During the Congress, mathematics was often not integrated with pedagogy in the presentations, showing that the intersection is empty.

The constructivism topic subgroups that I attended did not help me understand the topic any better. In one subgroup session, the two invited speakers were at odds on the meanings of several key concepts. When the discussant reacted, he held a third view! The presider brought early closure to the session when more than half the people got up and left. Sessions on knowledge and understanding, assessment, and the history of mathematics were informative and well done.

The personal interactions I had with old friends and new colleagues, along with the Howson lecture, were my high points. The Congress supplied me with another set of ideas that I will share with my peers in my workplace and various professional organizations. My rusty French enabled me to get along at the Congress and in the city. I urge all my peers to read the proceedings of ICME-7 and plan to attend future Congresses.

ICME-7

Arthur Johnson, II
Nashua Senior High School, Nashua, New Hampshire

My most vivid memory of ICME-7 is sitting in a large lecture hall with 500 other delegates listening to a professor from France talking about the role of geometry in modern mathematics culture. The lecture is in French, and I am wearing earphones, listening to a simultaneous translation in English (or, if I wished, Spanish). Beside me is a teacher from South Africa, behind me are two professors from Finland, and in front of me is a teacher from Hong Kong and another from Egypt.

ICME-7 had some resemblance to a large NCTM regional conference but with one pronounced difference. Before ICME-7 began, every delegate declared interest in one of twenty-three working groups. There was generally very little choice of speakers or venues within the working groups, sort of like ordering a prix fixe dinner. You had to go along with the menu. The working groups met for periods of 1.5 hours on each of four mornings of the conference. During the other times delegates were free to attend sessions, workshops, or demonstrations. This aspect of ICME-7 was most like a typical NCTM regional conference, with choices and freedom to attend anything that sounded interesting.

My working group dealt with student motivation. After opening talks and discussions on the first day, we divided into two smaller groups according to interest in theoretical or applied aspects of motivation. The two groups then met separately until the last day of ICME when we reconvened to share and combine ideas and conclusions. Each group was about seventy-five strong and contained a varied mix of delegates from many countries. The presenters in the applications group, for example, were from nine different countries.

I joined the applications group, and for each of the next three mornings we listened to four speakers who each delivered a 10–15-minute talk. We spent the remaining time in a question and discussion session with the presenters as a panel and the entire group participating. During the presentations and discussions, I was impressed with how much we had in common. No matter where a teacher was from, his or her concerns and experiences transcended the local environment or culture and were generally shared by all of us. Students, teachers, and teaching are basically the same all over, and what has been successful for one teacher is reasonable for another teacher thousands of miles away.

Our group defined seven areas that teachers should explore in order to motivate students:

1. *Use small groups* . The thought of the group was that students will take risks and find more interest in working with their peers than alone. Further, grouping students will give them the opportunity to teach their peers, another motivating force.
2. *Use real-life problems*. The utility and practicality of mathematics can do much to motivate students. If students can see a realistic use of the materials, they will be motivated to master mathematics.
3. *Teach the student not the subject*. The point here is that teachers can make mathematics easier or more difficult by their approach. Teachers should teach so that students can learn and enjoy some measure of success in mathematics.
4. *Create a supportive climate in the classroom to relieve mathematics anxiety*. A climate that engenders a feeling of security will encourage risk-taking. Such a climate will free students to express their negative feelings or experiences with mathematics. Once these feelings are revealed, they can be dealt with. One suggestion for relieving test anxiety is to help students relax with a recreational mathematics problem before any new learning experience.
5. *Present challenges that are not frustrating*. A multitude of problems exist that can challenge students. In the words of one presenter, "Anything new or surprising which is not frustrating is motivating."
6. *Show a void in mathematical knowledge*. It is human nature to want to have complete knowledge about some aspect of a topic. Teachers should give students a problem or situation where they

discover a gap in their knowledge. When this happens, the human tendency to want to "fill in" becomes a motivator.

7. *Use historical, recreational, and innovative problems.* These types of problems will motivate students by virtue of their novelty, unusual constructs, or their linkage to the human aspect of mathematics.

Naturally, each of the areas above is worthy of much more discussion, and the presenters did go into some details, supported in many cases by detailed handouts. What impressed me most was the discussion session after the presentations. All the members of the working group were generally of a similar mind about the problems associated with motivation and how these strategies would work with their students. In sum, ICME-7 was a full experience that gave me an opportunity to get to know educators from around the globe. Although we were from many different countries, we all spoke a common language, the language of mathematics education.

PRESERVICE AND IN-SERVICE TEACHER EDUCATION

Margaret Shoukry
Immaculate Heart High School, Los Angeles, California

It was exciting to interact with colleagues from around the world. I had naively assumed that the problems we in the United States have had with respect to math education were unique. Alas, this was not the case. Almost every session dealing with preservice and in-service education dealt with the frustration of finding people interested in teaching mathematics and the difficulty of equipping them with the skills that would make them effective in the classroom.

The attendees shared common concerns. It is imperative to provide mathematics education to all members of society. (This is especially true in third-world countries where lack of technological training severely restricts economic growth.) It is essential that teachers have the skills necessary to provide appropriate mathematics education, and it is incumbent on public and private agencies to provide the necessary training to teachers.

It was especially interesting that, although the speakers were from different continents and engaged in education projects directed toward very different target groups, they all shared a common thrust—namely, that quality teacher training requires lengthy field experience. That is, all of these programs, whether they were in France, Chile, South Africa, Israel, or the United States, had additional components beyond pedagogical coursework that included teaching in actual classrooms, supervised by master teachers. In some cases, the master teachers were part of the school site faculty; in most, however, they were teacher trainers from the preservice teachers' university training program.

Some programs were designed to begin after the completion of a degree; some incorporated the degree program with the teacher training component. The shortest program was four years (including degree), and the longest was six years. The focus of these mathematics teacher training programs was to insure that prospective mathematics teachers were familiar with both pedagogical skills and current mathematics curriculum trends. In addition, all these programs had components that offered in-service training to update curriculum information and to hone the teaching skills of experienced teachers.

Few speakers had follow-up data to offer to indicate the success of their programs. Most programs were still in their beginning years. But since all the programs reflected significant changes from previous programs that had not produced the desired results, all the presenters were hopeful that the programs would be successful. The presenters were concerned about budget problems that were threatening to undermine their programs, either in terms of reduced participation or decreased commitment to maintain the programs long enough to see the effect on student performance.

Despite the common problems that all mathematics educators seem to be facing, it was exhilarating to see the enthusiasm and commitment to improving the quality of mathematics education worldwide. It should be very interesting to follow up on these programs at ICME-8 in Seville in 1996.

ICME-7: PERSONAL REFLECTIONS ABOUT MATHEMATICS EDUCATION

Vânia Maria Pereira dos Santos
Indiana University, Bloomington, Indiana

The Quebec ICME-7 was well organized and provided opportunity for people around the world to share their ideas, problems, and hopes about mathematics education regarding teaching, learning, assessing, and researching. It was not possible to attend all the planned activities, but the opportunity for exchanging ideas with scholars from all over the globe was one of the exciting aspects of ICME-7. In particular, the happy-hour meeting was a nice break in the daily routine of the Congress and a chance to discuss our individual points of view and to raise questions about research studies in which we are involved. Many times this dialogue raised new questions about whether such investigations will improve the learning or teaching of mathematics or raised issues about curriculum development or teacher development.

This was my second opportunity to go to an ICME Congress. At ICME-6 I learned a lot and had the opportunity to make friends with mathematics educators from all over the world. In the last four years, I met some of these scholars and a few others from other countries at other conferences. In particular, my apprenticeship as a mathematics educator has evolved from 1985 to July of 1992. During that time, I received financial support from Brazil to participate in international conferences and to enroll in a doctorate program in mathematics education in the United States. In other words, the most important component of my lifelong learning experience as a mathematics teacher and a mathematics educator has been the richness of blending together my professional experiences in Brazil, my participation in mathematics education conferences, and my doctoral studies at Indiana University.

It was helpful for me to participate for the second time in Working Group 6 (WG-6): "Preservice and In-service Teacher Education." In the 1988 Budapest ICME-6, I shared my experience in Rio de Janeiro by identifying and analyzing preservice elementary teachers' difficulties in mathematics. In the Quebec ICME-7, Diana Lambdin (USA) and I shared our experience with alternative assessment in a mathematics content course for preservice elementary teachers that uses an innovative mathematics pedagogy (e.g., emphasizing cooperative learning; problem solving; alternative assessment; communication, writing, and reflectiveness). It helped to discover that other scholars in the world have been identifying similar problems while engaged in teacher preparation and have also been seeking for alternative ways to improve teacher preparation. However, because the WG-6 had so many subgroups, I felt that it was not possible in one week to come out with a more global view of the problems of teacher education at both preservice and in-service levels.

The lecture presented by Glenda Lappan (USA), "Training Teachers or Educating Professionals? What Are the Issues and How Are They Being Resolved?" offered a general view of the crucial issues in education.

It was very hard for many participants from other countries to follow the general ideas and examples presented by Mary Lindquist at the opening session of the WG-6. This was because of the (a) the difficulty some participants had with the fast way of speaking of Americans; (b) unfamiliarity with the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989), the *Professional Standards for Teaching Mathematics* (National Council of Teachers of Mathematics, 1991), and the call for reform in school mathematics in the United States and Canada; and (c) a lack of familiarity with the reality of American schools, media programs, and jokes that are clear to Americans. In short, for me, the overall presentation of Mary Lindquist was quite interesting because it reinforced several issues that I have been discussing at Indiana University. The essence of the content addressed in the two lectures, however, was clear only to audiences already familiar with the issues discussed in teacher education in the United States and Canada.

I strongly agree with Hatfield (1989) when he said, "We Americans have often appeared in the past to believe that our world leadership role means that others need to study and emulate what we are attempting" (p. 31), and unfortunately, I have to say that many times this is still a feeling for many of us who are foreigners when we listened to American speakers. In addition to the problem of nonnative English speakers having to understand Americans and American school reality, another problem that was raised in some

lectures dealt with the exclusion of references of studies or theories from other countries. It was questioned, "Why do American scholars include only American references and a few from England, Canada, or Australia and do not include references from a broad range of countries?"

It was very productive for me to participate in the workshop "Using Spreadsheets in Mathematics Learning," at the miniconference on "Computers and Calculators in the Curriculum, 11–16," presented by Portuguese mathematics educators. This was true because Brazil and Portugal speak the same language and the Brazilian school system still has many similarities with the Portuguese school system that was the base of the Brazilian educational system many centuries ago. The possibility of joint work between Brazilian and Portuguese mathematics educators should face fewer barriers than joint work with mathematics educators from other countries. There is a positive possibility of interacting with scholars from Latin American countries and scholars from Portugal and Spain that share similar languages (e.g., Portuguese and Spanish) and, in many cases, similar problems of teaching and learning mathematics and of preparing teachers to teach mathematics at all school levels. With these cases, there is a concrete possibility of planning joint efforts that can be effective because all the members share more or less similar views of the cultural, social, economical, and political issues that can foster or impede changes in education. Therefore, these scholars (who speak Spanish or Portuguese languages) can learn from each other by offering ideas from each other's particular educational reality.

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ICME-7 AND LEARNING TO TEACH MATHEMATICS

Glenda Lappan
Michigan State University, East Lansing, Michigan

Many things have happened in mathematics education worldwide since ICME-6 in Budapest. We have had in the United States in the four intervening years a period of consensus building around a vision of mathematics learning and teaching articulated in four documents:

- *Everybody Counts*, Mathematical Sciences Education Board
- *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics
- *Professional Standards for Teaching Mathematics*, National Council of Teachers of Mathematics
- *A Call for Change*, Mathematics Association of America.

There have been, of course, many other important mathematics education documents published during these four years that help articulate the direction for change in mathematics teaching and learning at all levels K–16. These four, however, would give the reader the heart of the vision of powerful mathematics for *all* children and the professionalism of teaching in support of this vision.

My particular focus and interest at ICME-6 and ICME-7 was to discuss with my colleagues from around the world the issues and promising interventions in the preparation and support of teachers of mathematics. Therefore, I was very interested in the opening address of the conference, which was given by Geoffrey Howson (UK). The title of his talk was "Teachers of Mathematics," and his premise for the talk was "teachers are central." It is on the shoulders of individual teachers within a country that the health of mathematics education rests. Howson believes that teachers should be seen as a national asset. He challenged us to help build worthwhile and satisfying employment for teachers or else we will not get what we want for our children. We should ask ourselves how proposed changes will help teachers and under what conditions. This clear stance on the importance of teachers supports and reflects the increasing attention of the mathematics education community to research and development on teacher education and teacher change.

Around 1985, the U.S. Office of Education funded the National Center for Research on Teacher Education (NCRTE) at Michigan State University. The NCRTE studies have helped us to begin to separate fact from fiction about both the students and the programs for preparing teachers in the United States. A summary of NCRTE research is contained in the NCRTE publication *Findings on Learning to Teach*. This publication asserts that majoring in an academic subject in college does not guarantee that teachers have the specific kind of subject matter knowledge needed for teaching. Another finding was that giving teachers knowledge about the cultures of various groups does not necessarily enhance their ability to teach children from these groups. These and other findings raise issues for the mathematics education community that need to be addressed.

The NCRTE has been re-funded for a second five years with a change of name, The National Center for Research on Teaching and Learning. The name change reflects a commitment to studying the complex issues of teaching, learning, and the interaction of the two.

Working Group 6 on "Preservice and In-Service Teacher Education" was organized by John Dossey (USA). The working group met four times for an hour and a half each time. The first and last sessions were meetings of the whole group. The two middle sessions were in small groups focusing on various aspects of teacher education. Mary Lindquist (USA), president of NCTM, opened the working group sessions by giving an overview of teacher education issues. She focused her talk on the NCTM *Professional Standards for Teaching Mathematics* (PSTM).

The PSTM, in the first section of the document, lays out a vision of teaching mathematics that is in support of the implementation of the *Curriculum and Evaluation Standards for School Mathematics* for all students. The writers of the PSTM identified four aspects of teaching that were judged to be so central to good teaching that they could form a set of standards about what teachers need to know and be able to do. These four aspects of teaching are (1) selecting *worthwhile mathematical tasks*, (2) orchestrating *classroom*

discourse, (3) creating an *environment* for learning, and (4) *analyzing* teaching and learning. The major question facing mathematics educators working with either preservice or in-service teachers is, How do teachers learn to make good decisions about tasks, discourse, environment, and analysis?

Diana Lambdin (USA), Indiana University, reported on the work she has been involved in at Indiana on assessment in mathematics classes for pre-service teachers. The intent of their assessment work is to create an environment for preservice teachers that mirrors appropriate instruction for elementary school children. The assessment framework for the course on problem solving includes writing in journals with specified and open prompts, self-reflection, and the making of concept maps. In addition, each test starts with a problem done cooperatively in a small group and then moves to individual work that revisits and extends the problem done as a group.

The University of Indiana project on assessment is a rare example of education faculty having the support of the Department of Mathematics in trying to improve the learning of mathematics for preservice teachers. The Mathematical Problem Solving course is offered in the Department of Mathematics and taught by mathematics graduate students. However, the materials and the assessment were developed by Diana Lambdin with Frank Lester and John LeBlanc, all from the College of Education.

My comments so far have reflected work going on in the United States. The important part of an ICME is the interaction among mathematicians and mathematics educators from around the world. Because my primary goal for the conference focused on teacher education, I found the final address to Working Group 6 by Claude Comiti (FRA) to be extremely interesting. Comiti gave her talk in French, which was a challenge for those of us from the U.S. However, we had two excellent interpreters and Comiti's English versions of overheads to help us understand.

Comiti reported on a major restructuring of Teacher Education Programs in progress in France. The restructuring is growing out of a concern for the depth of preparation and the kind of students attracted to preservice programs. A major goal of the restructuring is coherence and coordination across all aspects of the training—across levels from primary to secondary and between theory and practice. Students have three years at a university and then enter a two-year professional program at a teacher education special school, an IUFM, associated with a university. The two years of professional training in an IUFM give a student (1) general information; (2) information in the disciplinary field such as didactics, history of mathematics, and epistemology; and (3) experiences in teaching.

There are national competitive examinations at the end of the first year to select those who will be allowed to continue on to teacher certification. These tests include, for mathematics, two written exams, two oral exams, one math lesson, and a special test in which the candidate has to analyze a teaching situation. The competitive aspects of the program are helped by generous loans from the government for students preparing to teach mathematics or science. These amount to 70 000 francs per year, or about $14 000.

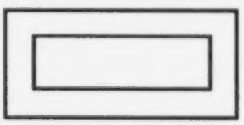
I left Quebec City, pleased to see the increased presence of teaching and teacher education in the international conversation. Although the weighting given to issues may be different from country to country, it is clear that the issues in teacher education are remarkably similar. I look forward to the next chapter in our progress worldwide at ICME-8 in Seville, Spain.

REFLECTIONS ON ICME-7, THE INTERNATIONAL CONGRESS ON MATHEMATICAL EDUCATION

Sharon Crumpton
Belmont University, Nashville, Tennessee

As a result of attending the Seventh International Congress on Mathematical Education, I grew professionally in three areas. There was a heightened awareness of the relationship between the beliefs that a teacher professes and the reality of attempting to implement those beliefs in the classroom. A thread that ran through many of the sessions was that teacher beliefs are often not translated into action. It is possible for a teacher to sincerely believe that certain teacher behavior is appropriate and even believe that such behavior actually occurred in the classroom; however, after observation, others often find that the desired behavior was not there. Sylvia Johnson (UK) reported on a project in which gender issues were addressed and where student teachers were then assessed on whether or not they treated male and female students equally in their practice teaching. Although students indicated that they had treated males and females equally, observers determined that such was not the case. Thomas Cooney (USA) reported that teachers say that they write questions that require deep understanding and thinking at a high level of abstraction; however, in practice such questions have rarely been developed. Elizabeth Jakubowski (USA) found that most teachers do not become researchers in their classroom, that is, they do not use their classroom as a place to learn about their teaching. Researchers reported few solutions to the discrepancy between teacher belief and teacher action. Accordingly, it was determined that research should be continued in the area of translating teacher beliefs into practice and that teachers need time to reflect on their teaching and then to be able to change.

Learning to pose problems was the topic of a presentation by Marion Walter (USA) who suggested that students be asked to write several problems based on a specific diagram such as the one below:



Michael Mitchelmore (AUS) suggested that students be asked to discover the maximum area of a rectangle with a given perimeter by using graph paper, scissors, and blocks, and Beth Southwell (AUS) suggested that role playing be used to simulate a teacher attempting to convince a principal to allow the use of calculators in the school. Southwell also suggested presenting the formula and theory to students and then having the students teach one another. Thomas Cooney suggested several ways to assess student understanding, for example, asking students to develop a figure with a given perimeter rather than just asking them to determine the perimeter of a given figure.

One of the most valuable aspects of the meeting was the time to talk with mathematics educators from around the world. When I sat with a professor from the Netherlands, I was pleased to discover that he had actually worked with the van Hieles. At the end of the excursion day, I accepted a kind invitation to tea from a professor from India, working in Africa. From her I learned about the status of women in Africa. The conference offered time to become friends with a professor who works at an institution in a neighboring state, and also time to renew friendships with people I have known for a long time.

These experiences have become an invaluable part of my professional growth and will result in several curriculum changes in my classes. I will never forget that for a few days I was part of a community that was much larger than any I had ever experienced.

CULTURAL DIVERSITY AND BONDING IN MATHEMATICS

Eileen L. Poiani
St. Peter's College, Jersey City, New Jersey

Discovering—and rediscovering—the cultural influences on the creation, teaching, and learning of mathematics are fundamental to the ICME experience through both formal sessions and informal dialogue with international colleagues. Nearly one-quarter of the working groups and topic groups at ICME-7 explicitly included issues of culture in their program descriptions, a significant increase over past congresses.

My cultural adventure began at the airport in New York by renewing an ICME-6 acquaintanceship with a professor from Egypt who had just published an article relating mathematics to culture. It continued with Working Group 10, "Multicultural and Multilingual Classrooms," which explored its charge through four subgroups: curriculum resources and materials, teacher education, curriculum for the twenty-first century, and language and culture in mathematics education of indigenous groups. This paper provides a glimpse of the group's examination of diversity in mathematics classes.

Integrating the richness of cultural diversity throughout the curriculum and classroom experience, rather than annexing it as a separate topic, was clearly the consensus of this working group. Connections to historic mathematical achievements by individual cultures as shown in the curriculum cards recently published by Claudia Zaslavsky (USA) help to recognize the mathematical contributions of distinct cultures.

More than ninety different cultures are represented in the northern New Jersey county where my institution, Saint Peter's College, is located. School enrollments across the country in rural and suburban areas as well as urban centers are becoming increasingly multicultural and multilingual. Working with students who communicate in English at school but speak a totally different language in the home challenges teachers across the globe from the United States to South Africa.

Working Group 10 looked at teaching strategies, both in preservice and in-service programs, that recognize cultural differences in learning styles and in the mathematical notions that children bring to class. For instance, in some cultures, notions of measurement or abstraction are not part of their basic experience or language. The situation is complicated by the fact that the teacher may not be a role model for the dominant cultures of the school.

New patterns of immigration to Sweden have led to more than eighty different languages being spoken by children in the major cities' elementary schools. To improve mathematics education, researchers in Sweden identified many different algorithms of arithmetic and algebra that these diverse cultures use. With these distinctions in mind, the teachers are creating tasks and curricular materials that are understandable across the cultures.

Some working-group participants explained their use of the case study approach to explore and to sensitize other teachers to the cultural differences in learning styles, approaches, and expectations. Assessment instruments also reflect what the culture deems important in mathematics and illustrate what measurements the culture values.

In the Netherlands, culture-specific problems are part of the new mathematics curriculum for students aged 12–16. Developed at the Freudenthal Institute under the direction of Jan de Lange (NLD), the problems provide a new conceptual context for learning mathematics and look at the same basic problem at different grade levels with increasing degrees of sophistication.

To test their theories at the middle school level, the Freudenthal Institute is engaging in an American-Dutch collaborative project with the National Center for Research in Mathematical Sciences Education headed by Thomas Romberg (USA) at the University of Wisconsin. Experimentation and evaluation are continuing for the duration of the project from 1991 to 1996.

In his plenary address, Geoffrey Howson (UK) spoke of the powerful influence of teachers as individuals and as a profession in each country. But despite the common bond of their concern for students and for mathematics, the cultural positioning of the teacher, the available resources, and the society's view of mathematics vary widely across the world.

If multicultural awareness is to occur in teacher preparation and in classroom implementation, then mathematics educators and the public will have to influence authors and publishers to incorporate into their textbooks meaningful examples and problems–and not just folklore–from diverse cultures. Until that occurs, the participants agreed, change will be slow and scattered.

Working Group 10 as well as Topic Group 2 on "Ethnomathematics and Mathematics Education" and Topic Group 5 on "The Social Context of Mathematics Education" considered the mathematical understandings of indigenous peoples and examined some curricula that were built from them in South America, Australia, Africa, and Asia. Since Ubiratan D'Ambrosio (BRA) coined the term "ethnomathematics" in the mid-1970s, interest in the subject has been growing steadily. From individualism in western society to an emphasis on uniformity in the East, from the influence of women in the home to their absence in many classrooms of the world, such factors emphasize the different cultural underpinnings from which mathematics is learned or taught.

Sessions on the "Popularization of Mathematics," the subject of a study by the International Commission on Mathematical Instruction, also showed that mathematics educators must work within cultural mores to promote public appreciation of our discipline.

In the United Kingdom, a Yorkshire television program called "Fun and Games" sought to entertain and increase awareness of mathematics with challenging mathematical puzzles and games. With humor and two highly competent game show hosts, the audience participated in an unrehearsed program to promote the attractiveness of mathematics and extend the mathematics culture. The British penchant for crossword puzzles translated into a three-year successful run of the twenty-five-minute show that reached over 10 million viewers.

Suited to another culture and audience, a six-part television series was created in France to look at topics such as mathematics in dance and space. The program focused on solving intellectually exciting problems in contemporary mathematics.

The ICME miniconference on "Calculators and Computers" offered strands by age cohort that dealt with topics from Logo to graphics calculators and from assessment to preservice and in-service teacher training using technology. Whereas the new National Curriculum in the United Kingdom stresses constructive and active use of computers and calculators for all children, struggling Third World countries lack adequate resources to purchase them, and the Japanese school system continues to frown on the use of calculators in the classroom.

Technology creates a new classroom culture and provides a unique opportunity for bonding in mathematics across cultural and linguistic lines. Students bring their own cultural and technical familiarity with mathematics to class; the presence of computers and calculators for interactive problem solving offers a common tool. Seymour Papert (USA), the inventor of Logo, noted that "The learning environment of the near future will be radically different." As computers shrink to no larger than a box of chalk, students and teachers can explore a variety of real-life problems that cut across disciplines and enable students to work alone and in groups. In a few years, children should be able to communicate with a "knowledge machine" to ask a question like, How do giraffes sleep? and obtain an instant response. The advantages and risks of such technology remain to be explored further.

Attention to culture instills a sense of history and fosters respect for cultural differences in the process of teaching and learning mathematics. The lessons of ICME-7 are fully in keeping with the action recommended in the National Research Council's *Everybody Counts* (1989):

> International studies of mathematics and science education . . . document that the social context of education has a greater influence on student performance than does practice. Parental attitudes, student expectations, and teacher self-respect are among the most significant factors in quality education and they depend uniquely on culture.

INNOVATIVE ASSESSMENT OF STUDENTS IN THE MATHEMATICS CLASSROOM

Winnie Miller
Hillsboro High School, Hillsboro, Oregon

The gentleman on my left was from Denmark, and the woman on my right was from Taiwan. As we conversed at lunch in such different accents, it was amazing to discover that although we came from far corners of the world, we still shared common concerns and questions about issues such as authentic assessment and gender equity. For the three of us, it was our first ICME conference and our first trip to Quebec. Later that same afternoon, I sat at a bench with Joan Countryman (USA), and in just a short time she reaffirmed my own philosophy about using writing in my mathematics classes. Burt Waits (USA) and Frank Demana (USA) suggested that I talk with Jim Fey (USA) about incorporating graphing calculators into the middle school curriculum. Later in the week, Ray Henderson (USA) would alleviate some of my anxieties about how portfolios might be used in classroom assessment. On another occasion, Penelope Flores (USA) promised to send me a rubric she had used for evaluating student projects. These encounters show that the fruitful connections to be made in attending conferences like ICME reside just as much in the small talk among colleagues as in formal presentations on the program. There is much to be gained in sharing with one another!

Glenda Lappan (USA) talked about worthwhile mathematical tasks and their use in integrating mathematical thinking with mathematical concepts and skills, in capturing students' curiosity, in inviting students to speculate and pursue their hunches, and in nesting skill development in the context of problem solving. Max Stephens (AUS) argued that assessment should consist of a broad range of activities that, taken together, reflect the objectives of the curriculum in a balanced way. Hence, investigative projects or expanded worthwhile tasks may be the best assessment tool. The English, Canadian, and Australian mathematics educators appear to be the furthest along in terms of creating assessment tasks. Scoring these is yet another issue.

Assessment work in Portugal has two major goals: to develop understanding of the role mathematics plays and to develop a generally positive attitude toward mathematics. Their mathematics curriculum is student centered and encourages group work, problem solving, thematic sequences, and open-ended projects. Some examples of their project work for pupils aged 12 to 15 include the following:

1. Studying the evolution of the number of children born in the last three generations (in the family of the student).
2. Designing an ideal classroom.
3. Building a panel of mosaics based on geometrical transformations.

The Portuguese have found that this practice also has its pitfalls. Money is always an issue. This kind of assessment is expensive and time consuming to administer and score. In addition, a project can be highly successful one year and be a disaster the following year, depending on the motivation of the group of learners.

Above all, I was pleased to hear, see, and experience for myself that my personal beliefs and practices about alternative assessment and other topics are also of interest to, and happening in, other mathematics classrooms, not only in the United States, but throughout the world.

ASSESSING: THE DISTANCE FROM RESEARCH TO PRACTICE

Patricia L. Duckhorn
Sacramento Country Day School, Sacramento, California

As a teacher, a K–8 mathematics specialist, and a regional in-service presenter for the Northern California Mathematics Project, I came to ICME-7 with nearly 3000 colleagues from ninety different countries looking for justification and support for present and changing practices and information on persistent problems. Although I heard of vast amounts of research being done in assessment from problem solving to strategies for preservice teachers, from questioning "questioning" to constructivist points of view, I was often amazed at the distance between what researchers say and how it is translated or applied at the classroom level. For me, this conference was a time of discouragement and excitement, frustration and hope.

As part of the Working Group on "Assessment," I was encouraged to learn of the many alternative strategies being developed and implemented in all parts of the world. Max Stephens (AUS), shared investigations done as large, end-of-term assignments by eleventh and twelfth graders. These activities were rich in mathematical content, provided multiple entries, and encouraged students to establish restrictions and set parameters before they could solve problems. Students presented mathematical arguments using word processors and graphing utilities, all in report format. The reports were impressive pieces of work. One of the positive effects of this program was the work's relationship to the regionally mandated tests. This relationship forced the teachers to change their classroom emphases in order to prepare students for the tasks. This changing assessment and teaching at grades 11 and 12 is also having a dramatic effect on teaching in grades 7 to 10.

Discouraging issues revolved around the amount of teacher time it took to both create and evaluate the assessment, the fear of discontinuation of funding, and tertiary institutions' perceived danger of non-standard assessment models. Many people, especially intermediate and secondary school teachers, felt angered by this lack of faith, especially when training, time, and financial support for these efforts in many areas of the world were limited, if not nonexistent. Even Stephens acknowledged the difficulty in mass education required by these changes. Yet despite some of the more discouraging discussions, it was very exciting to see what others were doing in the areas of curriculum and assessment change.

The Congress was an event of a lifetime for me. I saw the wonderful content-rich problems of the Netherlands, shared authentic assessment ideas with curriculum developers from Sweden and Canada, argued gender bias with a couple from New Zealand, and stayed up until the wee hours of the morning sharing teaching experiences with colleagues from about the world. Despite the mixed emotions evoked by some presentations, I was overwhelmed by the feeling that these forums were the place to solve, debate, and create new questions; that our world is really quite small; and that through the exchange of ideas and the camaraderie of colleagues, all things are possible.

ICME-7: AN ASSESSMENT

Evan B. Alderfer
Ocean County College, Toms River, New Jersey

The Seventh International Congress on Mathematical Education was held at Université Laval in Quebec in August 1992. The only other ICME I had attended was the sixth in 1988 in Budapest. There were a number of important similarities and differences. Both congresses were held at universities in cities with excellent gastronomic reputations. Both attracted more participants than had any previous ICME.

One of the great benefits of an International Congress is the opportunity to meet colleagues from around the world. For perhaps the majority, English is a second or a third language, so that a great effort must be made to speak and listen in a language that is not the mother tongue. Sadly, Americans seem by far the worst offenders in talking too fast, using slang and jargon, and failing to remember that many in the audience cannot listen as fast as the speaker can talk. Several presiders in sessions I attended had to ask speakers to speak more slowly. The offenders in each case were Americans. Several Americans, however, presented their papers in both English and French or spoke in English with overlays in French.

The working group I attended, "Innovative Assessment of Students in the Mathematics Classroom," divided into five subgroups for two of the four sessions. The sessions were all for the presentation of papers and were not really "working" groups because work was not done, except by the speakers. Friends who attended other working groups reported the same. I put my name and address on several lists in my working group just as I did in my action group in Budapest. I never received anything from the Budapest group and will be pleasantly surprised if I do from the Quebec group. The organizer of my group, Julianna Szendrei (HUN), did an excellent job, however, of sticking to the topic and getting others to do so as well. She spoke of the work of the group between now and 1996 when it will be scheduled to resume, but no mention was made of the difficulty of keeping 130 or more people from over twenty countries in touch with each other for four years with no money. Despite these difficulties, I found the working group interesting and informative.

In addition to the working group, I attended the two sessions of the topic group on "Statistics in the School and College Curriculum." These sessions were also primarily for the presentation of papers, but they were not supposed to be anything else.

Among other activities was a highly structured half-day miniconference on "Calculators and Computers." In each of five groups, we attended a plenary lecture, a workshop, and a presentation. Unfortunately, in the workshop I chose, the calculators were alleged to be held by Canadian customs, so that session, too, became a lecture.

The pleasure and comfort of the delegates were not ignored. The math trail in Old Quebec was well-trod by ICME-7 badge wearers. The opening reception at the Museum of Civilization in the Lower Town was pleasant but could never have measured up to its Budapest counterpart. The five happy hours, two of which provided snacks as well as beverages, were quite popular, despite the near impossibility of finding whomever it was one needed to find. As at ICME-6, there was an excursion day that provided a welcome break between heavy meeting days.

The mechanics of ICME-7 were as difficult to master as at ICME-6. It took me an hour each evening of reading, studying the program, and using the campus maps to plan my itinerary for the next day. At U.S. meetings of NCTM, MAA, or AMATYC, I have been able to do all of this planning before leaving home, thus making better use of my time at the meeting. On the positive side, the organizers in Quebec did a good job of staggering sessions at midday so as to lessen the lunch crush at campus eating facilities.

On rereading all my ICME-7 literature after returning home, I am amazed at what I missed but pleased with what I did. For any who may consider an ICME for the first time in 1996 in Seville, I would encourage attending. You will be with several thousand colleagues from around the world, some of whom will have spent over a year's salary to attend.

MATHEMATICS AS COMMUNICATION: A UNIVERSAL ENDEAVOR

Frances R. Curcio
Queens College of the City University of New York, New York, New York

"Mathematics as Communication," one of the unifying themes of the NCTM *Curriculum and Evaluation Standards for School Mathematics*, received much attention during the Seventh International Congress on Mathematical Education. Organized by Heinz Steinbring (GER), Working Group 7 was devoted to "Language and Communication in the Classroom." The interest, attention, and concern expressed by conference participants demonstrated the importance, on an international level, of examining and studying the different aspects of the language of mathematics for communicating and clarifying ideas.

At a plenary session, Steinbring described language as a cultural and social means of constructing new objects in the real world. A major problem in the study of mathematics is that meaningful mathematical knowledge is needed by students to understand, participate, and enter into the culture of mathematics. It is the role of the teacher to make understanding as easy as possible by employing language as an introduction to and a sharing of the culture. A broad question posed to the participants was, "What does a good or adequate mathematical language look like in general terms?" Three forms of language were described:

1. *Spoken* between and among students and teachers
2. *Words written* in texts, journals, diaries, notebooks, reports, and worksheets
3. *Symbols, formulas, graphs, tables, and models* that encompass specific elements of mathematics language

The focus of Working Group 7 was to examine "how understanding, comprehension, and meaning of mathematical knowledge develop in communicative teaching and learning processes." Attempts were made to identify the patterns, structures, conditions, obstacles, and opportunities in actualizing these processes. To facilitate discussing the different views, interpretations, and aspects of mathematical communication, the working group was divided into five subgroups looking at specific issues:

1. Construction of Mathematical Knowledge in Communicative Processes
2. Role of Language in Mathematical Communication
3. Social Patterns and Constraints in Mathematical Communication
4. Reading, Writing, and Discussing to Communicate Meaning
5. Role of Symbols and Diagrams in Mathematical Communication

During the Congress, the five subgroups met at the same time. Because I participated in Subgroup 4, this report contains a summary of the major points related to reading, writing, and discussing to communicate meaning in mathematics. The subgroup was organized and coordinated by Susan E. B. Pirie (UK). The presentations highlighted issues of meaning versus understanding by discussing the communication "of" mathematics and communicating "about" mathematics. The first of the two sessions focused on the former, highlighting hermeneutics and semantic distance between terms and expressions. The second session focused on the latter, highlighting journal writing, pupil-pupil discussion and problem solving, and communication strategies.

Communication "of" Mathematics

The subgroup presentations began with Maria Aparecida Viggiani Bicudo (BRA), who provided an overview of "hermeneutics"—deciphering and understanding the signs of humans' work. Particular emphasis was given to the role of the teacher in presenting subject matter via text material. On the basis of his or her knowledge of the subject matter, the teacher interprets, explains, and translates as he or she serves as a mediator between the text and students' processing of the content in the text. Implications for instruction include having students underline words they do not understand very well and having students consult a dictionary to begin to build meaning of unfamiliar terms and words. Although the study of hermeneutics

examines the relationship between reader and text, the participants agreed to focus on meaning between the deliverer and the receiver and not limit discussion to reader-text interactions.

In the second presentation, Antonio Marafioti Garnica (BRA) provided an example of hermeneutics, using a definition of *ordering relation* from a mathematics textbook. The procedure for text analysis to identify and underline obscure terms entails reading and reviewing the material by both the teacher and the student. At this point, reference is made to a dictionary. The student then rewrites the text in his or her own words, using diagrams and analogies. Questions and ideas related to the rewritten text emerge. The degree of text comprehension can be determined by the natural language employed by the reader in rewriting the text. The process is interactive among reader, text, and teacher.

A. Gagatsis (GRK) discussed *added meaning* by referring to the notion of *semantic distance* between terms and expressions. The semantic distance is dependent on the "space of meaning," "a set of expressions or terms," and a "set of given codifications" (i.e., mappings). The degree of confusion about the meaning of a given term or expression is represented as the semantic width of the term or expression (i.e., a comparison between what the teacher or text means in relation to the student's interpretation of the term of expression). An example of an infinite semantic width was discussed. The ideas presented suggest a way of analyzing text and discourse and explaining differences of meaning.

Communicating "about" Mathematics

Ann Oaks (USA) and Barbara Rose (USA) discussed "Writing as a Tool for Expanding Student Conception of Mathematics." The questions that organized their presentation included the following:

1. What do our students know and understand about mathematics?
2. What do we want our students to know and understand about mathematics?
3. What are the benefits of using writing to learn mathematics?
4. What kind of writing can help achieve these goals?

Expressive writing activities that can be used in mathematics classes were discussed. These included journals, autobiographies, focused in-class narratives, explanations, notes, and process writing. There are benefits of writing as a tool for students as writers, teachers as readers, and the reader-writing interaction.

"Writing as a Vehicle to Promote Sense-Making in Mathematics and a Learning Community" was presented by Martin Hoffman (USA) and Arthur Powell (USA). Journal writing entails writing a diary of personalized experiences and feeling about mathematics, rather than keeping a log of activities. Spelling and grammar are not important for this activity and should not interfere with writing about one's feelings and experiences. Working with underprepared students, the authors analyze journal entries according to three dimensions: affective-cognitive, reflective-nonreflective, and personal-nonpersonal.

Recognizing the limitations of student-teacher dialogue in getting students involved in reading and communicating about mathematics, the authors introduced the idea of having students read each other's journals and share their feelings and ideas about mathematics. This process helps in building a learning community.

On the basis of student surveys and journals, the authors found evidence of student growth depicted by responses that move from nonreflective to reflective statements. Students described how reading the journals of their peers contributed to and influenced their own understanding, appreciation, and sense-making of the mathematics. The question was raised of how to develop more sensitive instruments to measure the three dimensions for classifying the types of journal writing that students produce and substantiate students' impressions of this process.

Pupil-pupil and pupil-text interactions were described by Frances Curcio (USA) in her paper "The Effects of Small Group Interaction on Graph Comprehension of Fifth Graders," which she coauthored with Alice Artzt. The overall focus of their work was "How do we help students to arrive at a deeper meaning when reading graphs?" The research question was "What are the effects of small group interaction on the graph comprehension of fifth graders?"

Definitions of key terms with examples were provided: *graph comprehension* was defined in terms of levels of comprehension—reading the data (i.e., literal), reading between the data (i.e., interpretation, interpolation), and reading beyond the data (i.e., prediction, extrapolation). *Cognitive behaviors* involve "doing" and "can be exhibited by verbal or nonverbal actions that indicate actual processing of information." *Metacognitive behaviors* involve monitoring, evaluating, and thinking about the task at hand, and "can be

exhibited by statements made about the problem-solving process." The theoretical framework for the study was provided by Artzt and Armour-Thomas (1992) and Curcio (1987).

The analysis of a six-minute videotape of fifth graders working on a graph comprehension task was illustrated using the framework of Artzt and Armour-Thomas. The patterns of behavior of the small group reaching a correct solution to the graph comprehension task mirrors the cognitive-metacognitive behaviors of successful problem solvers described in the problem-solving literature. A question for future examination is, How does the success of small-group interaction transfer and contribute to individual student's improvement of levels of graph comprehension?

Judith Fonzi (USA) and Constance Smith (USA) presented their paper entitled, "Communication in a Secondary Mathematics Classroom: Some Images." This paper developed out of the "Reading to Learn Mathematics for Critical Thinking" project funded by the National Science Foundation. The authors presented three questions that guided the presentation:

1. In what ways does communication take place in a mathematics classroom?
2. What might the content of the communication be, and how does it influence meaning-making?
3. What role does the learning environment play in supporting or maintaining positive communication?

Fonzi described her course, Math Connections, and the learning environment. Among the messages that were conveyed to the students were these:

1. We are a community of learners, we all come with important things to contribute.
2. Everyone benefits from sharing ideas and decisions and reflecting on experiences.
3. Mathematics is a human construction and an integral part of life.
4. Learning requires active involvement.
5. The teacher is not a disseminator of information—teacher and students work together to make decisions.

Students' opinions and beliefs about mathematics were surveyed on the first day of the class. All experiences were shared and discussed throughout the course by students and teacher. Reading and writing were an integral part of the learning. The teacher had very clear goals in mind and the materials were chosen carefully for the messages that they conveyed. The six-day unit included Davis and Hersh's *The Mathematical Experience* (1981) and the Project Mathematics! videotape *The Theorem of Pythagoras* (1988). The students were involved in different modes of communication: discussing, reading, writing, observing, showing and telling, modeling, and drawing. The configurations of communication included: student-self, student-student (primary mode), student-teacher, teacher-self, teacher-student, material-student. Students talked about technical mathematics, the nature of mathematics, the process of doing mathematics, feelings about doing mathematics, and learning—both the content and the process. The findings identify a complex integration of all aspects of these modes and configurations.

Concluding Comments

Each of the papers generated a rich discussion. Unfortunately, there was an inadequate amount of time to address all the interests and concerns of the participants. In particular, the appropriate use of a dictionary, the problems that are caused by listing words and terms out of context, the power of symbolization, the difficulties encountered by bilingual students, the need to align assessment techniques with instruction, and the perceived intent of the didactical contract in terms of the roles of the teacher and students, were some of the issues that dominated the discussions. The participants and presenters left the sessions with more questions than answers.

Before closing the session, Gagatsis posed the question, "What is good communication?" Several participants suggested that any communication is good—just have students communicate. Other participants took exception to this remark because the quality and the nature of the communication may not reflect the desired on-task behavior (i.e., students may be discussing nonrelated interests or issues). Sometimes the teacher may present a task that is inadequate to promote small-group interaction of all students in the group (i.e., the task is appropriate for an individual rather than a group). Other reactions and ideas are too numerous to capture in this brief report. The work of the subgroup raised many issues,

concerns, and questions. There were enough ideas and questions generated to keep everyone busy for the next four years—at least!

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MATHEMATICAL COMMUNICATION IN THE CLASSROOM

Marta Civil
University of Arizona, Tucson, Arizona

ICME-7 provided an intellectually rewarding opportunity to interact informally with colleagues from different parts of the world. I participated in Working Group 7—"Language and Communication in the Classroom."

The general theme of WG-7 was the analysis of real communicative processes, where language is an important vehicle for the social construction of mathematical knowledge. Bernadette Janvier (CAN) addressed the issue of what kinds of interventions teacher educators might consider in shaping the education and practice of preservice teachers. We, as mathematics educators, want changes, but are we effectively communicating this to our preservice teachers? Many of us have reached the convictions we presently hold as a result of an evolution over time. Are we sharing our own evolution with our students? Janvier was concerned about the messages that preservice teachers may take with them from our classes. They may wonder if what we are suggesting is really "better." This notion of disassociation between what mathematics educators believe doing mathematics is and what the outside community believes was also addressed in the topic group I attended.

Susan Pirie (UK) talked about the many different forms of communication in mathematics (e.g., ordinary language, symbolic language, visual language). She then focused on the difficult links between language and understanding. *We cannot assume* that if a child can do a mathematical task, then this child has the language to talk about it, and what is more important, that if a child cannot express himself or herself in a mathematics context, then he or she does not understand.

For the next two days the members of WG-7 worked in subgroups. I was in the group focusing on "the role of language in mathematical communication." The four papers presented in this subgroup were related, yet diverse enough to allow for a productive discussion. My paper presented samples of eight preservice elementary school teachers talking mathematics. I addressed the question of what can be inferred about these preservice teachers' understanding of and beliefs about mathematics. I focused on the interplay of language, understanding, beliefs, and a social component in a small group discussion approach to the teaching and learning of mathematics. I raised the question of how "mathematically incorrect" we should allow students' expression to be for the sake of communication. I also wondered about what the preservice teachers took away with them from this emphasis on talking mathematics.

Patricia Morgan's (UK) paper presented samples of students' written reports in a mathematics class. The reports shows a variety of explanations for the same task. Morgan pointed out the difficulty of how to evaluate such explanations. Teachers' beliefs about what constitutes mathematical activity are likely to influence their evaluation of these reports. Morgan's presentation is particularly relevant to mathematics educators who work on assessment alternatives and on the use of writing in mathematics. She raised the question of what forms of language are appropriate to communicate mathematical ideas in particular social contexts. She also wondered to what extent teachers' and students' beliefs and knowledge about appropriate mathematics language were the same.

Rainer Loska's (GER) paper presented a teaching approach that follows a modified Socratic method where the teacher's role is that of an organizer of discussion among students. Loska observed that this approach to teaching may make some teachers uncomfortable because of the variety of understandings that students are likely to bring to a task. Loska's questions revolved around the issue of when and how the teacher should intervene in the students' discussion. He also raised the question of how to teach this method of instruction to preservice teachers.

Megan Clark's (NZ) paper addressed the relationship between the context of problems and the students' gender. Her presentation focused on what kinds of context and language appeared to be more meaningful to female students. She raised the questions of whether one may be encouraging dependence in learners by posing problems in "friendly" contexts and of how far one should go in doing this. In the topic group we

addressed a similar issue about the need to make mathematics culturally relevant to students. Some participants expressed their concern that in trying to make things too relevant, we may end up teaching different kinds of mathematics to different ethnic groups and making it hard for ethnic minorities to compete.

The questions that the presenters raised served as a framework for the work in the subgroup. The main conclusions reached by the group were the following:

1. We need to give good, challenging, and fruitful problems to engage students in discussion. The problems must be seen as problems by the students. If the students feel ownership of the problems, they may develop a better sense for what needs an explanation. A concern that many of us shared is that the students often are not sure of what and when to explain and of what is a mathematical explanation.
2. To help students improve their communication in mathematics, we could emulate the writing process. Students would go through several drafts of their mathematics work, revising and editing them. There was some discussion of a French research project where they are working on teaching students how to communicate properly in mathematics. Some of the participants interpreted that project as trying to teach the *official discourse in mathematics* and perceived it as different from their view on communication in mathematics.
3. Teacher education occupied much of our attention in this subgroup. There was some concern that preservice teachers may think that the message is to "just" let students talk. However, is any talk okay? Preservice teachers may feel overwhelmed and anxious about letting their students go in different directions when discussing a mathematics task. Sierpinska (POL) pointed out that we should not "make the prospective teachers believe that *constructivism* means lack of control of the constructed meanings." To help teach preservice, or practicing, teachers how and when to intervene in a classroom discussion, someone suggested videotaping the classes for further analysis with teachers.
4. Interestingly, in both the working and topic groups, we discussed a similar possible solution to help alleviate some of the teachers' anxieties. In the topic group, the issue was that the teacher may perceive as a burden the need to know about the different cultures present in the classroom. The proposed solution was to view these culturally diverse children as resources. The teacher should count on them as cultural informants. In the working group, the issue was that the teacher may feel overwhelmed by all the different ideas the students may bring to a task. By letting the students be resources to each other, they can provide explanations and contribute to their peers' and their teacher's understanding of mathematics.

SOME REMARKS ON ICME-7

Floyd L. Williams
University of Massachusetts, Amherst, Massachusetts

Of approximately 3500 persons gathered for ICME-7 in Quebec City, I was among the few research mathematicians in attendance. Mathematics educators have been divided, unfortunately, by perception, practice, and other factors into three main groups: research mathematicians, mathematics educators, and teachers. This division has been to the detriment of mathematics as a whole—as has been pointed out already in noted reports such as "School Mathematics in the 1990's" and H. Greudenthal's book *Revisiting Mathematics Education* and as was pointed out in the opening plenary address to the Congress by G. Howson (UK). In addition to Howson's outstanding address, the Congress featured three other very fine plenary addresses by M. Klawe (CAN), C. Laborde (FRA), and B. Mandelbrot (USA).

Among some twenty-three working groups with their various subgroups, I was involved during the week with Group 15 on "Undergraduate Mathematics for Different Groups of Students" and in particular, with Subgroup 4 of this group, which focused on "Mathematics for the Specialist Teacher or Researcher." The three other subgroups of Group 15 dealt with the teaching of mathematics to nonspecialists, mathematics for business, and research in postgraduate mathematics education.

Our subgroup session began with enlightening overview remarks by Lynn Steen (USA). He spoke of the decline in the number of mathematics majors in the past fifteen years and the recommendations (for the United States) that resulted; on goals to increase the specialist student's knowledge of mathematics, with a move toward the achievement of parity for women and minorities; on the general need for improvement in the quality of teaching and learning, including change in campus climates toward a holistic approach in research and pedagogy and university links with secondary schools; and on standards for secondary level teachers. It was recommended, for example, that the first two years of a major should consist of core courses (some of which might spill over into the third year): calculus, differential equations, probability and statistics, and linear algebra. The third and fourth years might involve certain unspecified electives but at least some course in depth (a full year of real analysis, for example) and courses with applications and connections to other mathematics or to other fields, such as physics, economics, or computing.

Emphasis was placed also on (1) the support of individual students (advising, with follow-through efforts, and the building of student self-confidence) and (2) formal structures for departmental renewal (departmental self-evaluation and consultant review, periodic curriculum dialogue, etc.).

Steen's remarks, which set the tone, were followed by remarks (during the week) from Ronald Brown (UK), Man Keung Siu (HK), Harley Flanders (USA), Jean-Pierre Kahane (FRA), and Adaremi KuKu (NIG). Brown's remarks dealt with a new course developed by him and Tim Porter (UK) on mathematics in context (versus content as the staple of traditional courses). Kahane spoke with profoundness and elegance on the basic need of the specialist to think, to write, and to read. The researcher must learn to think about what he is doing, how what he does relates to other fields, the social and cultural meaning of his work, and how it lines up with that of the "masters" in the great scheme of things. One's reading ought to include historic and original papers and perhaps papers in physics (to some extent). Kahane pointed out the need, in general, for the researcher and the student to "have eyes and ears in many directions" and to therefore have means of communication with many persons in many areas.

On the final day, the four subgroups of Group 15 were brought together for a summary discussion and report presentations. My Subgroup 4 has in its report a discussion of the following: (1) problems and issues, (2) goals for an undergraduate training program for specialist mathematicians, and (3) assessment. One of the goals in (2), for example, is the encouragement not only of independence, creativity, self-confidence (which I mentioned earlier), and technical development but also an appreciation of professional issues such as (a) the place of mathematics in the scheme of things (consistent with Brown's remarks on mathematics in context), (b) the methods of working in the subject, such as thinking, learning, planning, writing and reading (as Kahane advises), calculating, evaluating, communicating, and (c) appreciation of some contemporary research developments, problems, and issues. (2) and (3) are linked; realization of the

goals discussed in (2) depends crucially on the assessment process, which should have sufficient variety to assess adequately the achievement of these goals.

The Congress, which I attended as a member of the United States Commission on Mathematics Instruction, afforded me the unique and special opportunity to speak with an array of people ranging from fourth-grade elementary school teachers to graduate students to academic deans—people with a common desire to continue their own educational growth and to foster that of their students. In conjunction with my initial remarks, I would hope that the delineation lines for research mathematicians, mathematics educators, and mathematics teachers would soon soften or disappear and, in particular, that more of the so-called research mathematicians would embrace a broader view of mathematics education (at all levels) and allot some of their time (at least a small measure thereof) towards involvement in such a Congress and in some of the vital educational issues of the day.

MY DIARY OF ICME-7

Susan L. Ganter
California State University, San Bernardino, California

The task of presenting a summary of ICME-7 for the benefit of my colleagues who were unable to attend is an overwhelming one. I have spent much time mulling over different ways in which to organize my thoughts on the conference and have come to the conclusion that it is impossible for any one person to cover everything. Therefore, I have decided to simply report my six days in Quebec City in a journal format. Some entries from my "journal" follow.

Luckily, we arrived a day early and therefore had time to recover from traveling. It was also a great opportunity to wander about the beautiful Université Laval campus. I was amazed at the incredible diversity of the 3500 participants from places as far away as New Zealand, Israel, and Japan! ICME-7 was indeed an International Congress, and I was now more excited than ever to meet and talk with participants in the working groups the next day.

The opening reception and happy hour was an event to behold! A huge crowd of international mathematics educators from every imaginable background gathered for free drinks and a feast of sandwiches and munchies. We were very impressed with the effort made by the conference organizers to create a festive atmosphere that made it possible for old friends to reacquaint and new friends to meet. We could have made dinner out of this party but were anxious to venture into Old Quebec for a meal. So we piled into a taxi with some new friends and went exploring. The conference could not have been located in a more charming city with the cobblestone streets and (definitely) French atmosphere. It was hard to decide where to dine because cafes that poured incredible aromas into the street were all around us. However, we finally settled on a quaint restaurant with a view of the St. Lawrence River.

My conference sessions began with an introduction to Working Group 3, "Student Difficulties in Calculus." This plenary session, led by David Tall (UK), was designed to give an overview of the present research and teaching problems in the field, as well as to outline the objectives of the subgroups that would be formed for the next two sessions. Each of the organizers of the five subgroups gave a brief summary of the goals of their group so that participants could choose the subgroup that they would attend the following days.

After this plenary session, I attended a lecture by Ronald Lancaster (CAN) entitled "Mathematics and the World around Us." Lancaster presented an interesting array of problems designed primarily for the secondary school student. However, the problems were such that they could be adapted to other educational levels, making the talk applicable to most of the diverse audience. For example, a problem on the McDonald's corporation could ask something as simple as "How many Big Macs can you buy with $5.00 if one costs $1.89? to something as complex as "What price for a Big Mac will generate the most profit given data on sales over the past ten years?" Unfortunately, this was just one of seven lectures presented during this time slot and was chosen on the basis of a very broad title that caught my eye. I feel certain that I missed other talks that were also very interesting and would have found it easier to choose if the lecture titles had included a brief description of the content.

The afternoon consisted of the introductory session of Topic Group 12: "Graduate Programs and Formation of Researchers in Mathematics Education." I chose this session because I thought it would consist of ideas and pointers for developing a graduate program in mathematics education, something that my department is currently planning. However, the session had an entirely different, but equally interesting, emphasis. This first meeting of TG-12 contained several presentations from different countries on the requirements for a Ph.D. in mathematics education. It was fascinating to learn just how similar, yet different, the programs in various cultures are. For example, most programs require about one year of full-time (or two years of part-time) study past the bachelor's degree in courses such as learning theory, teaching methodologies, and pure mathematics. This year of study is customarily followed by three to four years of research with a faculty member, resulting in several papers and culminating with the writing of a dissertation. Of course, there were variations even in these requirements, since some programs allow their

students to "customize" their degree to fit their needs. However, the major differences between the programs of different countries seem to occur in their administrative organization. Specifically, countries such as France have nationalized programs that are developed and approved by an appointed committee, whereas the United States and Canada have hundreds of individual programs developed at universities that have no required guidelines; rather, they develop their programs on the basis of a set of "understood" standards. Although the results are fairly similar, the power of individual faculty members in these different countries is very different, and each method has both its strengths and weaknesses.

The second meeting of TG-12 consisted mostly of a discussion of the results of an international survey of the graduate programs in mathematics education at sixty-one universities around the world. Data on the general characteristics of each program were collected, including information about the faculty, students, course content, and research requirements. A report of the results was made available to each of the topic group participants and a list of attendees and their addresses was developed so that communication within the group on ideas for graduate programs in mathematics education could continue.

ICME—IS IT FOR PRECOLLEGE TEACHERS?

Cynthia Lanius
Milby High School, Houston, Texas

Before I relate my perspective on ICME-7, I'd like to provide a context that will make it more meaningful. I teach mathematics at Milby High School, the largest school in Houston, Texas. Our school population is about 3600, 80 percent Mexican-American. I say this with pride. I am proud both of my school and of my profession. When I first heard of the Congress, I immediately wanted to go. But then I began to wonder if I should. Now from my perspective of having attended, I'd like to address the bigger question: Should precollege teachers attend ICME? Can they both benefit and contribute enough to make their going worthwhile?

The most obvious truth about a conference such as this is that it is impossible to do everything you'd like to do. I participated in the "Preservice and In-Service Teacher Education" working group, and the subgroup "Ethnomathematics and Teacher Education." I work with the Rice University School Mathematics Project as a Master Teacher and director of an in-service project. Also, I teach underrepresented minorities and was especially interested in studies related to minorities. I think the most meaningful experience of the conference for me was sitting in a classroom with about ten educators from Australia, South Africa, and the United States discussing equity issues. Interestingly, we found we shared similar experiences and philosophies in spite of the miles and borders between us. Every speaker emphasized the importance of the target group's involvement in the design of any program, rather than the university paternalistically imposing a program. Several of us exchanged addresses to continue the dialogue.

At a conference like this, many sessions blend into an aggregation. I attended sessions of Psychology of Mathematics Education (PME) and International Organization of Women in Mathematics Education (IOWME). I felt it unfortunate that so often these met concurrently and I had to choose between them. I saw special projects designed to facilitate girls' learning of mathematics. Here again I was able to make contacts that could be extended so that I could make use of those projects in my own work. At another time I got to hear for the first time fellow Texan Uri Treisman (USA) report on his important work with minorities. I spent one afternoon watching math videos, one by Caleb Gattegno and one with Georg Polya. I have one word written in my notes about each: for the first, *breathtaking*; for the second, *brilliant*. Throughout the week I met leaders from the United States and other countries that I feel gave me a better perspective of mathematics education. Language at an International Congress is a problem that is not always effectively solved, and such was the case here. At one lecture, the line for headphones was so long that the session started before many could obtain them. One session in our working group was given in French and translated into English. Not the best. Yet, all in all, this was a minor distraction. There was plenty going on in English.

Now back to the original question, Should precollege teachers attend ICME? I believe it is an extremely valuable professional experience for teachers. No, they won't get teaching activities to use in their classrooms, but they will get a broad sense of mathematics education and their role in it. That is how they will benefit; now how can they contribute? It was interesting to me through the week that when people learned of where I teach, they often had many questions. As one man put it, you're out there in the trenches. Teachers should even be encouraged to present papers. Many teachers are active in developing projects and should be the ones helping to report on them. Precollege teachers should have an integral role in these conferences. To ignore them in this process, to discount the work of such an important segment of the mathematics education community, is wrong. We have four years to develop strategies to provide for this at the next conference.

MATHEMATICAL MODELING IN THE CLASSROOM

Bart Braden
Northern Kentucky University, Highland Heights, Kentucky

My primary interest is in college-level mathematics, so I presented a talk at the miniconference on "Calculators and Computers in Mathematics" (Strand MC-4: "Undergraduate Mathematics"), took part in the working group on "Mathematical Modeling" (WG-14), and attended what I thought would be a related topic group (TG-8: "Teaching Mathematics through Project Work"). In addition, I spent several hours examining posters, software, and videotapes and listening to invited addresses. In the following comments, I elaborate on some of my impressions of specific events.

The working group on "Mathematical Modeling" was very successful. Although everyone seemed to be in agreement that problem-oriented curricula are becoming better established and increasingly successful, I was impressed by Jan de Lange's (NLD) reminder of continuing obstacles: problems in assessment, the loss of some good features of traditional teaching, student objections to open-ended problems, loss of structure in the mathematics curriculum, difficulties involved in teacher retraining and associated support services, and so on. The talk by Ronald S. Lewis (CAN), describing his new high school course "Fractals in Your Future," was interesting and very appropriate for the session on modeling.

A description by Heinrich Abel (GER) of a video instructional program on feedback control technology, produced by a consortium of universities in Germany and the United States, was perhaps the most valuable event of the entire conference for me. The working group format gave me the opportunity to introduce myself to both Lewis and Abel and to talk with them at some length after their talks. Such opportunities to make contact with colleagues in other countries were exactly what I hoped to find at ICME, and the organizers of WG-14 are to be congratulated for arrangements that provided these opportunities.

My only disappointment with the ICME-7 program was with the topic group on "Teaching Mathematics through Project Work." Here the discussions were on very general themes on the first day, and thereafter focused only on primary and secondary level experiences. Another flaw in the conference was the absence of a significant contingent from the People's Republic of China. China has an enviable record of success in teaching mathematics at all levels, and efforts should be made to make it possible for others to learn of their methods.

THEORIES OF LEARNING AND THE THEORY AND PRACTICE OF PROOF

Susanna Epp
DePaul University, Chicago, Illinois

This is the second ICME I have attended—the first was in Budapest in 1988. On the basis of my experiences, I would recommend ICME attendance to almost any mathematician interested in exploring how people learn mathematics and how teachers can best help them do so. Why do I say "almost"? Because, as with any large meeting, not everything proceeds smoothly, descriptions of sessions may not be adequate to make the best choices, and presentations include chaff along with wheat. But for those who can tolerate imperfection, there are many rewards.

The "Theories of Learning" working group brought together a wide range of mathematicians and mathematics educators having an interest in cognitive psychology and psychologists with an interest in mathematical learning. Perhaps it was the mathematician in me that was especially impressed by Giyoo Hatano's (JPN) presentation. Practical precepts for mathematics education were deduced from a comprehensive set of basic principles of knowledge acquisition, established through the research of cognitive psychologists. Other presenters explored additional, sometimes competing, sometimes supplementary, and sometimes overlapping, theories of learning in mathematics with the aid of developing a common theoretical framework.

The topic group on "Theory and Practice of Proof" included a lively panel and follow-up subgroup sessions. Some presentations were interesting as existence proofs—for instance, descriptions of successful programs in France, Germany, and Italy in which 11- to 13-year-olds are introduced to proof and refutation in informal ways that make such activities natural and meaningful. Bob Davis's (USA) description of a class of ordinary fourth graders in New Jersey who were led to engage in mathematical thinking of the highest order is evidence that such activity is possible on the home front. My only criticism of the working and topic groups was that insufficient time was provided for group discussions. The emphasis on formal lectures seemed at odds with the ostensible commitment of many Congress organizers to interactive learning.

The "Calculators and Computers" miniconference was well organized and informative. The tertiary-level instruction session included plenary presentations on the use of computers in teaching discrete mathematics and differential equations. The workshop on teaching calculus using Derive was particularly relevant to me because I have been doing that recently.

Often poster sessions play a very minor role at meetings, but at ICME-7 they seemed to occupy a more important position. Posters were displayed in high traffic areas so that there was a constant flow of people reading and discussing them. The hours reserved for authors to meet with those interested in their work were well attended and lively. The only unsuccessful venture was the poster roundtables. The idea was excellent: to give an opportunity for presenters whose work concerned a common theme to come together for discussion among themselves and with other interested parties. But it appeared that the choice of presenters to address a particular theme had been made capriciously.

The plenary lectures I attended were of generally high quality. Indeed, it was frustrating not to be able to fit more of them into a very full schedule. I would particularly mention those of Michael Closs (CAN) on Mayan mathematics, Jan de Lange (NLD) on comparing curricular change in the U.S. and the Netherlands, Zal Usiskin (USA) on exploring the meaning of "mathematics for all" in today's world, and Maria Klawe (CAN) on how she presents what it's like to do mathematics to school children.

One of the most important aspects of ICME-7 for me was the opportunity it gave for interaction with a variety of different people who, though they had come from all over the world and possessed many different perspectives, shared a common dedication to the process of education. It is encouraging to know that one is in such good company.